Femtosecond laser inscribed tilted gratings for leaky mode excitation in optical fibers

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***Abstract*— We consider the excitation of leaky mode resonances (LMRs) in an optical fiber that originate from light coupling initiated by small angle (<21°) tilted fiber Bragg gratings (TFBGs), through sub-micrometric grating inscription using a femtosecond laser. The femtosecond laser inscription triggers the interaction between the fiber's guided mode and backward coupling to the LMRs, a process that requires tight control of the grating period, reflection angle, order and the strength of the modified refractive index. The TFBG itself exhibits very low spectral polarization dependence, whereas the LMRs display strong polarization dependence in specific wavelength ranges, allowing for easy mode selection and opening avenues for various applications related to sensing and optical filtering. The tailored-for-LMR-generation optical filters were characterized for torsion, temperature and strain, exhibiting a very large negative strain sensitivity of –2 pm/με and a temperature sensitivity of 14.73 pm/°C that when compared with the grating response led to an exceptionally large strain response and remarkably well-behaved conversion matrix for the separation of strain and temperature in a single measurement. The LMR also proves to be insensitive to bend. The nature of LMRs is considered and confirmed by measuring their radiation field using an infrared camera, simulating their resonance wavelength using a finite element method and by experimentally observing their behavior, while changing the ambient refractive index.**

***Keywords*: Fiber optics sensors, Optical design and fabrication, Fiber Bragg gratings, leaky mode resonances, leaky modes, guided mode resonances.**

I. INTRODUCTION

A leaky mode is a mode that decays as it propagates in a waveguide. In fibers it can be simply described as a mode below the operating cut-off and is characterized by having an effective index below unity. To understand leaky modes, we must first examine their source. Leaky waves have been of great interest in the last decades in research areas of microwave engineering. They have shown promise in the concept of near-field stratified guiding structures due to their ability to provide controlled and directed radiation [1]. They were first discovered in the spectrum of an optical metallic reflection grating when illuminated by constant spectral intensity, in the form of anomalous sharp amplitude variations in 1902 by Wood[2]. In 1930, Fano recognized that some of these anomalies rise from forced resonances related with guided modes of the gratings[3]. These resonances are basically constructive interferences, which in the case of a grating correspond respectively to a guided mode of a stratified structure and the continuum of radiation modes of free space. A more detailed analysis in Wood's anomalies in terms of scattering resonances were explained in detail in the work of Hessel and Oliner, in the concept of leaky wave theory[4]. Their analysis considered an ideal periodic grating. Nevertheless, it correctly predicted and described important properties that were observed (i.e. effects from different incident polarizations). A more accurate analysis of Wood's anomalies has been recently developed, which takes into account the realistic geometry and material properties of optical gratings[5]. Through the years a plethora of research applications came from the study of leaky waves. Tamir *et al.*,[6] used structures that can support leaky modes as a medium to enhance the Goos–Hanchen shift; the lateral displacement when light is totally internally reflected due to the power flow associated with the evanescent waves excited at the interface. For specific incident angles and frequencies that guarantee phase matching between the incident wave and a leaky mode of the structure, a large portion of the incident energy penetrates the structure and is guided laterally as a leaky wave. Considering periodic gratings, a negative shift can be observed in homogeneous slabs with negative permittivity due to the excitation of backward-going leaky waves[7]. Resulting from this effect[8][9], light through a prism can couple to a metal interface, supporting leaky surface-plasmon waves. To this aim, the Kretschmann prism configuration[10] remains the basis of the majority of the recently developed plasmonic sensor devices[11][12] for liquid and gas detection[13][14]. Furthermore, in recent years leaky modes have also been exploited to produce total absorption in lossy layered media[15] and filtering effects in periodic slab waveguides[16][17] when combined with diffraction gratings. These guided-mode resonances (or leaky-mode resonances) are resonant peaks/dips in the reflection/transmission spectra, explained in detail using rigorous coupled wave analysis[18][19]. Such

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resonant effects have been studied extensively in recent years to generate several functionalities[20], such as broadband and narrowband filters and polarization control[21].In this work, we demonstrate the controlled excitation of leaky mode resonances (LMRs) in cylindrical optical fibers. The LMRs originate from backward coupling, initiated with a tilted fiber Bragg grating (TFBG) having an appropriate tilt angle, from which the guided propagating core mode backward couples to the leaky modes[22]. The controlled coupling to leaky cladding modes is a rare sight, since their excitation is very challenging to achieve. It has been previously reported in the case of long period fiber gratings (LPFGs)[23] and extremely tilted FBGs (ex-TFBGs)[24][25]. It is crucial though to state that these coupling methods excite primarily forward propagating modes. The here-examined TFBGs were fabricated using a femtosecond laser, via the plane-by-plane (Pl-by-Pl) inscription method [26][27]. In our case, the tilt angle is small, and the cladding mode coupling is generated exclusively in the backward propagation direction. The fact that the higher order leaky mode resonance is present, can be traced to the backward coupling of the TFBG, from the phase matching condition; and the nature of the inscription process, resulting from controlled period/order of inscription and the strength of the modified refractive index in the fiber core. The paper is structured as follows. We begin with a comprehensive analysis of the leaky wave theory for slab waveguides to obtain the relation for excitation of guided mode resonances. We then consider the generation of leaky modes in optical fibers through the work of Snyder *et al.* and correlate this with the generation of higher order cladding modes with the means of a TFBG. Finally, examine the sensing capabilities of these modes and offer confirmation of their nature through simulation and experimental methods.

II. Theoretical Background

A. Leaky wave theory

In any waveguide, the field distribution that satisfies the boundary conditions can be represented as a simple superposition argument defined in terms of inhomogeneous waves[28]. These waves can be best described as a set of discrete modes (eigenmodes) that correspond to poles in the transverse-wavenumber complex plane. In a closed guiding structure, the eigenmodes have typically purely real values for a propagating wave with constant amplitude or purely imaginary values for an evanescent wave with constant phase. If we consider losses, i.e. in an open waveguide (a waveguide with its dominant mode is bound; it can become a leaky mode when asymmetry is introduced), these modes become complex solutions of the eigenvalue equation. A "leaky" waveguide mode, as it is recognized, is characterized by a complex wavenumber with attenuation constant that is due to radiation loses[29]. However, depending on the polarization, this introduces multiple solutions to the dispersion equation (1) that leads to a *complex propagation vector*:

$$k = \beta - i \cdot a \qquad (1)$$

where β and α are, respectively, the *phase and attenuation constants* with i the imaginary unit. Moreover, when periodicity is introduced to the structure of the waveguide (more layers) the complex solution becomes Bloch waves. The single, real mode leads to an infinite number of spatial orders, with the imaginary part being exactly the coupling loss[17]. These modes have unique characteristics; the wave's phase velocity is faster than the speed of light and they can be radiated at specific angles depending on β. The smaller the attenuation constant, the larger effective apertures and therefore the narrower beam patterns [30]. In particular, leaky waves are inhomogeneous waves, propagating in lossless media has a Poynting vector that is parallel to the phase vector, causing the leaky wave to rise upwards from the guiding structure[31]. Two different types of leaky waves can radiate depending on the phase variation the propagation axis; forward leaky waves (β>0), leaky modes that radiate along the energy-flow propagation and are strongly attenuated. Backward leaky waves that radiate to the opposite direction (β<0) and are usually supported by structures with negative permittivity (i.e. plasmas), or by non-uniform periodic structures that excite the waves[20]. The direction of propagation is a leaky mode characteristic of paramount importance. In structures that can support leaky waves, the introduction of a phase-matching element, such as a diffraction grating or prism, induces the aforementioned guided/leaky mode resonances that can be observed in either reflection or transmission (Figure 1a,b). The resonances observed from the diffraction orders of a complex stratified structure have a phase component multiple of 2π:

$$\beta = k_0 n_s \sin\theta_i - \frac{2\pi m}{\Lambda} = k_0 (n_s \sin\theta_i + \frac{\lambda m}{\Lambda}) \qquad (2)$$

where $k_0 = 2\pi/\lambda$ is the wavenumber in free space, n_s is the refractive index of the surrounding medium, θ_i is the incident angle, m is the diffraction order and Λ is the period of the diffraction grating. Since the mode propagation constant of the waveguide is $\beta = (2\pi/\lambda)\, n_{eff}$ and n_s is equal to 1 (air), the coupling relationship to a guided/leaky mode resonance in this type of structure is expressed by equation (3)[19][32]:

$$\theta_i = \sin^{-1}(n_{eff} - \lambda * m/\Lambda_G) \qquad (3)$$

where λ is the wavelength position, n_{eff} is the effective index of the waveguide, m is the diffraction grating order and Λ_G is the period of the diffraction grating. The incident angle is the factor which defines the generation of leaky modes and its wavelength.

B. Leaky modes in optical fibers

In optical fibers as in slab waveguides, light transmission can be expressed as a composition of bound (discrete) and unbound (continuous) modes. Over a small wavelength range, bound modes undergo total internal reflection, representing the energy guided through the waveguide, while unbound modes undergo partial reflection, representing in turn the radiated energy[33]. Leaky modes are an excellent estimation of that radiated energy. It is considered as a bound mode below its cut-off frequency, i.e. a mode that has complex solutions for its eigenvalue equation, as aforementioned, which attenuates in the direction of propagation. All the above are described in detail in the work of Snyder *et al.*, demonstrating that whereas in slabs rays undergo refraction and attenuate rapidly, in a cylindrical waveguide these rays are sufficiently skew to the fiber axis and can propagate over long distances[34]. This leads to the definition of two groups of leaky modes. The “refracted leaky rays” undergo refraction in the core cladding boundary but exhibit no evanescent region in the cladding, whereas the skew rays are labeled “tunneling leaky rays” because they exhibit a form of electromagnetic tunneling at the core cladding interface due to the optical fiber’s curvature[35]. These rays propagate across the evanescent region and they leak extremely slowly compared to the refracting leaky rays. Moreover, their leakage vanishes as the wavelength value approaches zero. At this point we note that the radiation origin point and coupling to leaky modes and their classification are directly related to the higher order modes[36].

C. Concept of tilted FBG coupling to higher order modes

Tilted FBGs, like uniform FBGs, are created when a photo-inscription process occurs along the propagation axis of the fiber core, introducing a periodic and permanent modification of its refractive index (RI). In addition to the core mode coupling at the Bragg wavelength, the tilted refractive index modulation allows the coupling to cladding modes as well[37]. The size of the tilt defines the direction of the cladding mode propagation[38]; small tilt values yield backward propagation whereas larger values enable forward coupling. The excitation of higher order cladding modes (via forward propagation) with an extremely TFBG was first reported by Zhou[39]. In the coupling to cladding modes, the phase matching condition can be expressed as shown in equation (4)[40]:

$$\beta_i = \beta_j \pm \beta_G \qquad (4)$$

where β_i and β_j are the propagation constants of a mode while β_G corresponds to the grating wavevector. Using the TFBG coupling condition equation (4), we represent the effective refractive index of each mode with its corresponding wavelength position based on backward propagation, as follows.

$$\lambda^j{}_{clad} = \frac{[n_{eff,co} + n^j{}_{eff,clad}] * \Lambda_{TFBG}}{n_{order} * \cos(\theta)} \qquad (5)$$

where λ^j_{clad} is the wavelength of the j^{th} cladding mode resonance, $n_{eff,co}$ and $n^j_{eff,clad}$ are the effective refractive index values for the core and the cladding mode, respectively, Λ_{TFBG} is the period of the grating, n_{order} is the grating order and θ is the tilt angle. Figure 1c shows a classic representation of the coupling from the core mode to cladding modes in a TFBG. Figure 1a,b show schematics of structures that are capable generating leaky/ guided mode resonances. Similarities between planar configurations and TFBGs (Figure 1c) can be found, but in the first case the LMR generation is primarily related with the incident angle. In the latter, one can consider that each individual plane of a TFBG corresponds to a single layer of a multilayer stratified structure. The only difference between the two is that instead of having an external diffraction grating to induce the coupling to the LMR, we use the tilt of the internally written FBG to achieve coupling.

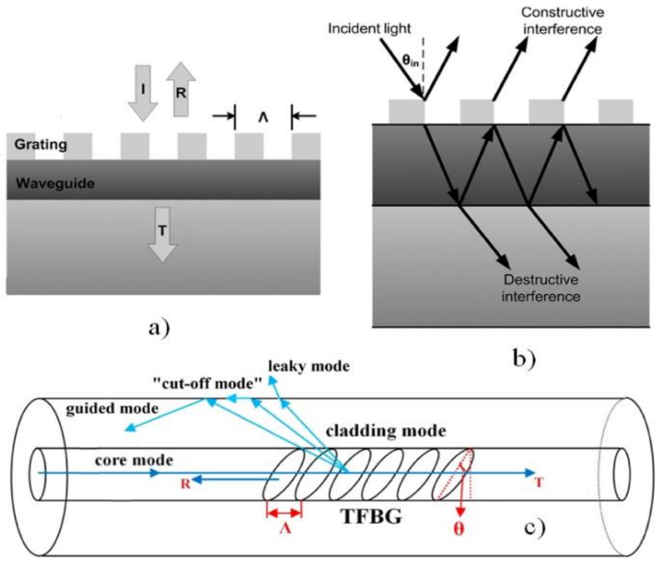


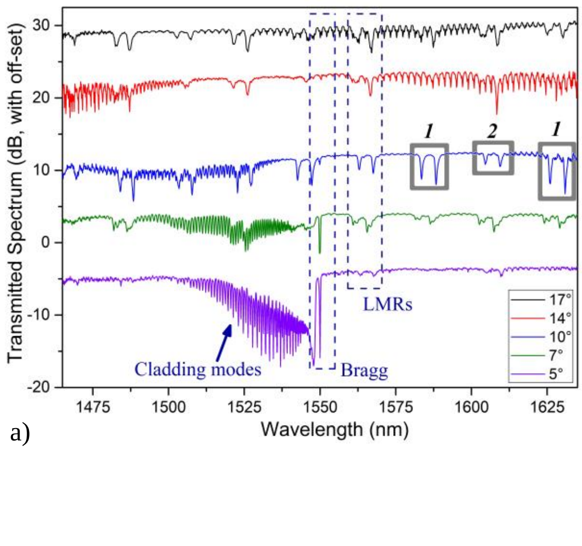
Fig. 1 a) Schematic view of a leaky/guided-mode resonance, b) Leaky coupling at resonance according to the model in ref. [41], c) Schematic of TFBG geometry. Coupling from the core mode to numerous cladding modes where R (reflection), T (transmission), Λ (grating period) and θ (Tilt angle) are shown.

III. TFBG Femtosecond Laser Inscription

TFBGs were inscribed in FiberCore photosensitive single-mode optical fiber using a direct-write, femtosecond laser inscription method (Pl-by-Pl technique), which offers flexible, through the coating inscription that allows for control of all the important TFBG inscription parameters, from which the LMRs are generated. A femtosecond laser system (HighQ laser femtoREGEN) operating at 517nm and pulse duration of 220 fs, was focused with a long working distance objective x50 (Mitutoyo) inside the core of the optical fiber. Suitable translation stage motion ensured that the inscribed planes had appropriate width (minimum 800 nm), angle and 3-dimensional refractive index change[42][43]. The grating period could be readily controlled for sub-micron changes in period and the grating order was readily controllable for second order and above (e.g. 10th-order gratings). In order to characterize the optical components, transmission spectrum measurements were made. The TFBGs under test were interrogated using a combined broadband light source (BBS) and polarizer, followed by a polarization controller and an optical spectrum analyser (OSA Advantest Q8384) for wavelength dependent polarization measurements. In order to better understand of the LMRs' nature several tests were made for 10th order 1 cm-long tilted gratings with different tilt angles. To investigate polarization behavior, we use a Luna optical vector analyzer (OVA CTe) to record the polarization dependent loss (PDL) spectrum, an alternative way to distinguish the presence of LMRs with greater resolution[44]. The measurement response to physical measurands was undertaken as follows, for temperature and strain measurements, respectively. First, the samples were placed in a climate chamber where the temperature was controlled with an accuracy of 0.1 °C with incremental steps of 10 °C from 20 - 100 °C. For the latter, samples were mounted on a translation stage with a 10 μm accuracy, whereupon strain steps of 100 με from 0 to 1800 με were applied. For bend measurements we introduced controlled compression on one axis, whereas rotation and torque were applied by using high-resolution rotation controllers, on the two and one fibre end, respectively.

IV. RESULTS AND DISCUSSION

In Figure 2a we observe that the tilt angle does not affect the position of the leaky mode resonance, whereas the cladding modes shift to the blue as the tilt angle increases, as expected. Based on the intensity of each LMR pair we can say that two sets of leaky mode families are excited; these are "families" that originate from different diffraction orders, as can be observed in Figure 2a, illuminated by a broadband light source and no polarization control. These families can be distinguished from the difference in intensity, indicated by the numbers 1 and 2 in Fig 2a. That signifies the importance of the tilt angle in the excitation process of



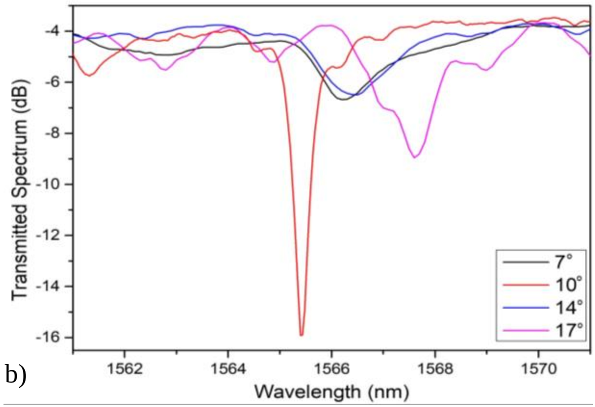


Fig. 2 a) Transmission spectrum of tenth order TFBGs at different tilt angles. For all cases the Bragg peak was selected at 1550nm, b) Comparison between different tilt angles with respect to transmission loss.

leaky modes resonances, as aforementioned. This is even clearer when considering Figure 2b. When polarization control is introduced, we observe a significant and strong notch in the TFBG spectrum, with an impressive ~15 dB dip for the 10° TFBG, as observed in Figure 3a. In the inset of the same figure, we show the polarization dependence of a set of leaky modes, which has a preference to one of the two orthogonal polarization states. Therefore, the 10° angle TFBG will be focused in the following measurements. At this point, we must note that higher order gratings

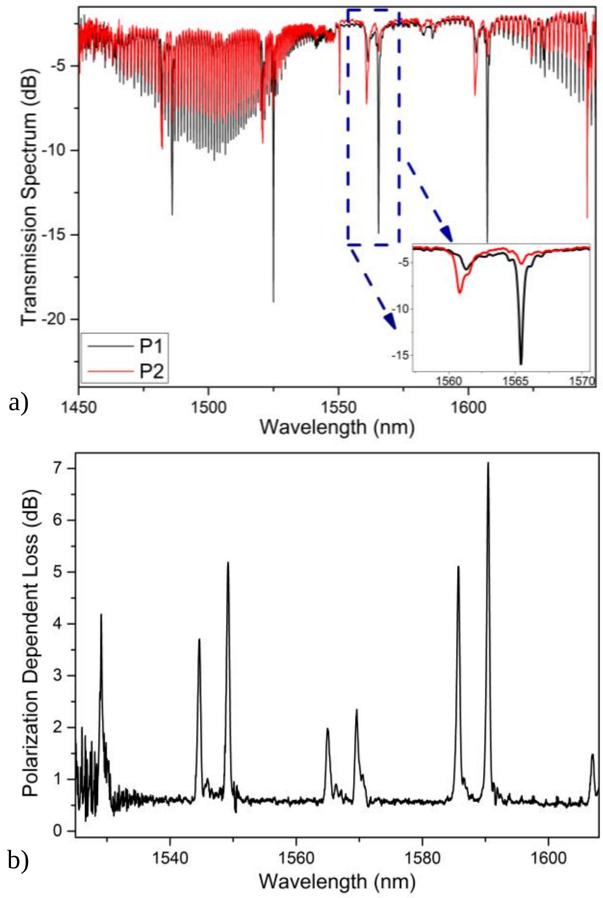


Fig. 3 a) Transmitted spectrum of a 10th-order 10° TFBG in the wavelength range 1450–1650 nm. Inset: Orthogonal polarization dependence of the leaky mode resonance closest to the Bragg peak. b) Polarization Dependent Loss of the 10th-order 10° TFBG in the wavelength range 1520–1610nm measured by LUNA OVA.

have less PDL[42]. Figure 3b confirms that we have achieved a largely polarization independent spectrum, apart from the LMRs that have polarization dependence at specific and selective wavelengths. To characterize the leaky mode resonances, measurements for temperature and axial strain response were made. For both cases, comparison of the sensitivity of the Bragg mode and its closest leaky mode resonance is displayed in Figure 4. The leaky mode resonance had a sensitivity of 14.73 pm/°C (K_{T2}) and –2.0 pm/με (K_{E2}) for temperature and strain, respe-ctively; whereas the Bragg mode had a sensitivity of 10.65 pm/°C (K_{T1}) and 1.2 pm/με (K_{E1}), respectively.

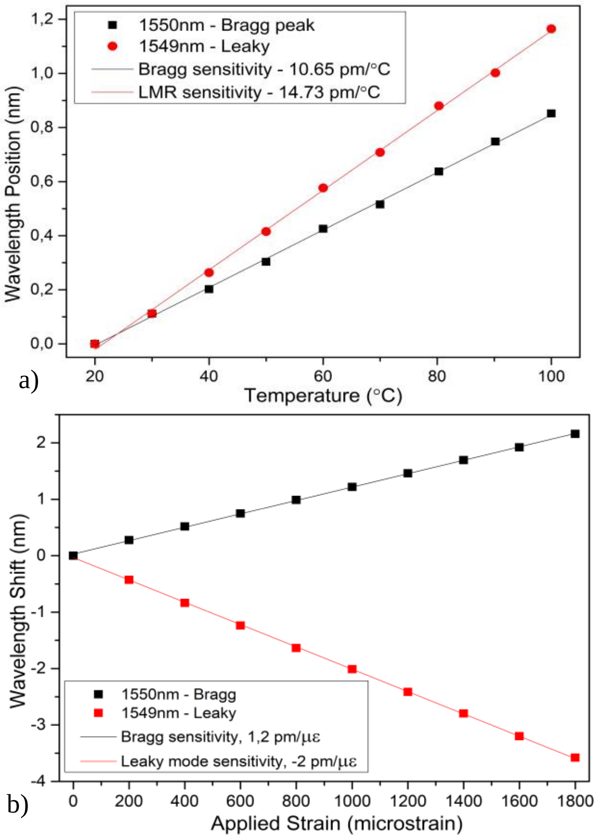


Fig. 4 Comparative measurement of Bragg mode with the closest leaky mode resonance in: a) Temperature, b) Axial Strain.

This results in a differential temperature sensitivity of 4.08 pm/°C and 3.2 pm/με differential strain sensitivity, almost three times larger than the Bragg mode sensitivity[45]. In order to decouple temperature and strain induced wavelength shifts with a single wavelength measurement, one can utilize the matrix equation (6), if the relevant temperature and strain sensitivities of two fiber modes are known (Bragg and LMR). If the matrix determinant is not 0 that means the sensitivity coefficients K are slightly different [46]. In this way, we can separate the temperature and strain response and quantify the errors that may arise from the measurements.

$$\begin{bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \end{bmatrix} = \begin{bmatrix} K\varepsilon_1 & K_{T1} \\ K\varepsilon_2 & K_{T2} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon \\ \Delta T \end{bmatrix} \quad (6)$$

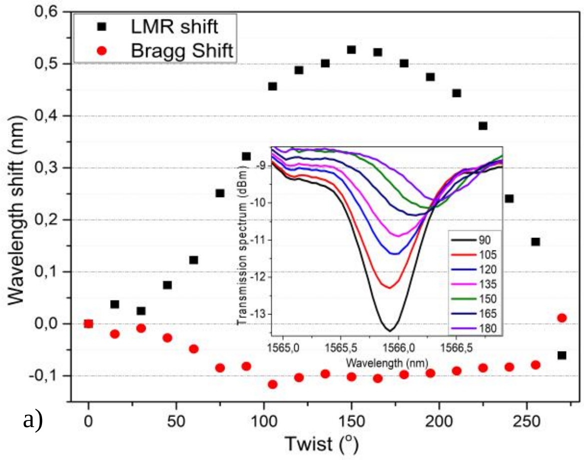
Where Δλ is the wavelength shift, K_T and Kε are the temperature and strain sensitivities, respectively, while the subscripts 1 and 2 refer to the Bragg mode and the leaky mode resonance, respectively. The coefficients were computed as follows:

$$K\varepsilon_1 = 1.20 \pm 0.0075 \text{ pm/}\mu\varepsilon$$
$$K\varepsilon_2 = -1.99 \pm 0.0070 \text{ pm/}\mu\varepsilon$$
$$K_{T1} = 10.65 \pm 0.1094 \text{ pm/°C}$$
$$K_{T2} = 14.73 \pm 0.1736 \text{ pm/°C}$$

The ratio of the temperature to strain coefficient for the Bragg mode is 8.87 με/°C and –7.36 με/°C for the leaky mode resonance. This exceptionally large difference in coefficient ratios allows us to calculate the errors for strain and temperature, following Brady *et al.*, in ref. [47]

$$\Delta\varepsilon \max = 0.47 \frac{\mu\varepsilon}{\text{pm}}, \qquad \Delta_T \max = 0.06 \frac{°\text{C}}{\text{pm}}$$

This is the best conditioned inversion matrix and lowest reported conversion error, to date. We also note that the extreme polarization dependency of these modes can be used effectively as a torsion sensor element. Figure 5a displays the wavelength shift during twisting, for which an amount of strain was unavoidable during the twisting action, as the fiber was marginally pre-strained during the mounting process in the manual rotators. We observe that there is a negative shift in the Bragg peak, meaning that the fiber is relaxing whereas the LMR red shifts. We can separate torsion and strain since we acquire the axial strain sensitivity from both the Bragg peak and the LMRs. It is known that the Bragg peak is not affected by torsion. Therefore, we calculate the applied strain from the Bragg peak and from that the effect of axial strain in the LMR's wavelength shift. Figure 5b shows the *pure torsion effect* on the LMR. In the inset of the Figure 5a one observes that in addition to the wavelength shift the leaky mode resonance is losing (rejecting) power, which is effectively coupled to its complementary polarization for the first 90 degrees exchange between the two polarizations of an LMR, giving an alternative way to examine the torsion sensitivity *directly*.



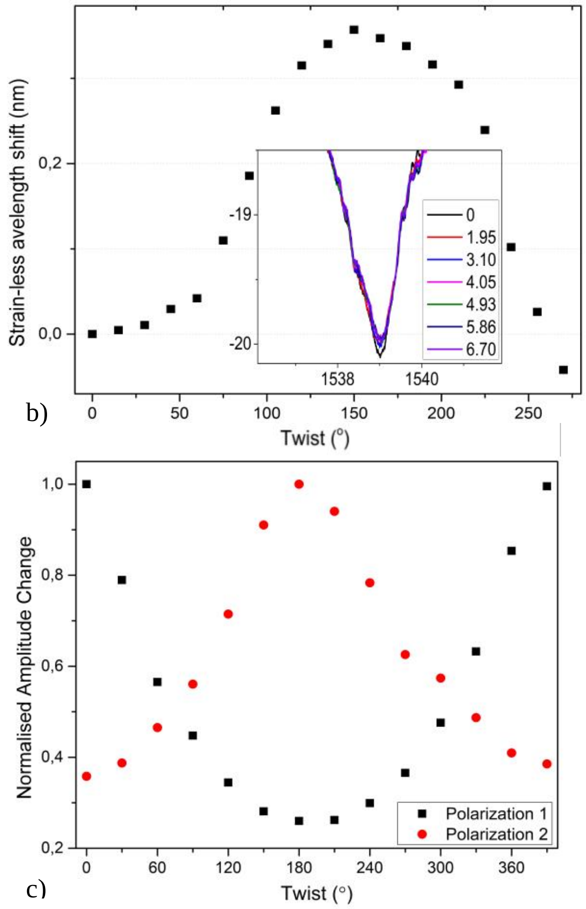


Fig. 5 Torsion behavior of a leaky mode resonance. a) Wavelength shift during twisting and from strain during the measurement. Inset: Actual mode shift and power de-coupling for the first 90 degrees, b) Pure torsion sensitivity of the LMR. Inset: Bend insensitivity per curvature change c) Normalized amplitude shift between the two polarizations of an LMR.

To make sure that all our previous measurements were not affected by accidental bend of the sample, thorough bend sensing measurements were undertaken. Using the rotation measurement setup we introduced compression with a translation stage, to evaluate the curvature of the device (Figure 5b, inset). The device response is completely insensitive to bend.

Simulations, observations and visual confirmation

We use the Fimmwave mode solver software, which includes the possibility to introduce transparent boundary conditions, in our simulations. The boundary condition is of great importance when simulating leaky modes due to their complex nature. The effective index of each higher order cladding mode was calculated using equation (5), with the leaky modes shown to have refractive indices below the unity[48]. Using equation (3) we simulated the possible wavelength positions of leaky mode resonances for a slab waveguide leaky wave structure in the case of a diffraction grating for different incident angles, with period identical to the inscription parameters of the sample under study in experimental results (curved). Figure 6 depicts a correlation between these calculations and the Fimmwave simulation (bar chart). It can be noted that the location of the modes and the

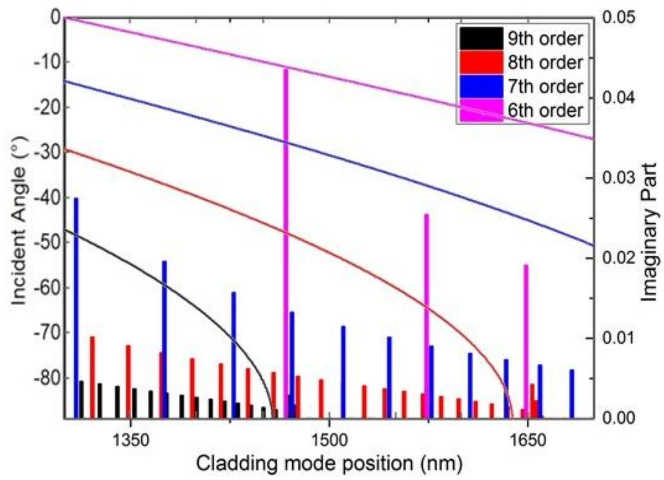


Fig. 6 Fimmwave simulation of the higher order mode location (bar chart) for each individual order along with the Matlab simulation (curved lines) for a leaky waveguide coupling as calculated from equation 3.

order coincide for both simulation methods. One simple way to verify that the effective index of these modes is indeed below unity is the fact that the leaky modes are present after the inscription process with the coating intact. Nevertheless, when the fiber is stripped of its acrylate jacket in the inscribed region, some resonances are enhanced, and others radiate; based on their respective effective refractive index. Additionally, we can ensure that all of modes disappear by immersing the TFBG in ethanol. Similar behavior was observed in ref. [49], where modes reappeared when the surrounding refractive index increases far beyond the individual mode's effective index. It is interesting to note that modes with effective indices below 1 they fall into the category of anomalous dispersion[50].

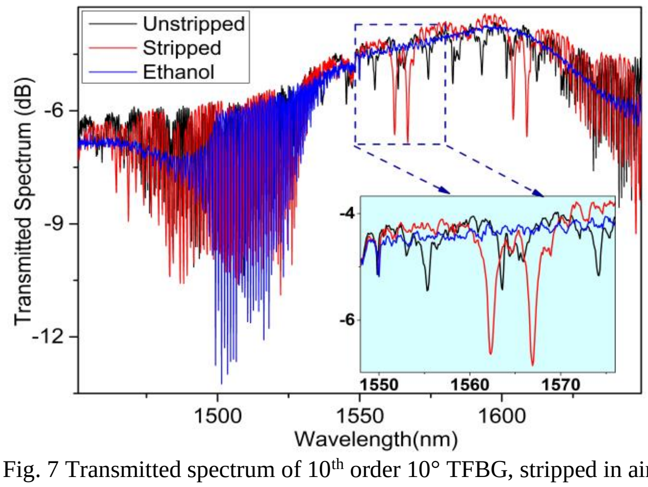


Fig. 7 Transmitted spectrum of 10th order 10° TFBG, stripped in air and dipped in ethanol. In ethanol, leaky modes vanish.

Figure 7 shows a comparison for the case of different surrounding refractive indices. We infer that the generated leaky modes belong to the group of the aforementioned tunneling leaky rays, since they exhibit evanescent behavior and interact with the surroundings. To further confirm that these dips in the transmission spectrum are leaky mode resonances, we imaged them using an infrared camera observation system. In this way, we record the wavelength-dependent light coupling and examine the behavior of these modes across the surface of a stripped optical fiber. We launch light into the fiber core using a HP-8186F tunable laser and tuned selectively to wavelengths coincident with the appearance of LMR dips in the transmission spectrum. In Figure 8a and 8b images captured by the infrared camera show the coupling in and out of a leaky mode resonant wavelength. Figure 8c shows a pixel intensity analysis indicating the change in power distribution. As expected, when tuning to a wavelength that is not coincident with the LMR, the power is confined in the core, whereas tuning to a wavelength coincident with the LMR results in the core losing its peak intensity whereas the fiber's surface "lights up" as the modes rapidly leaks.

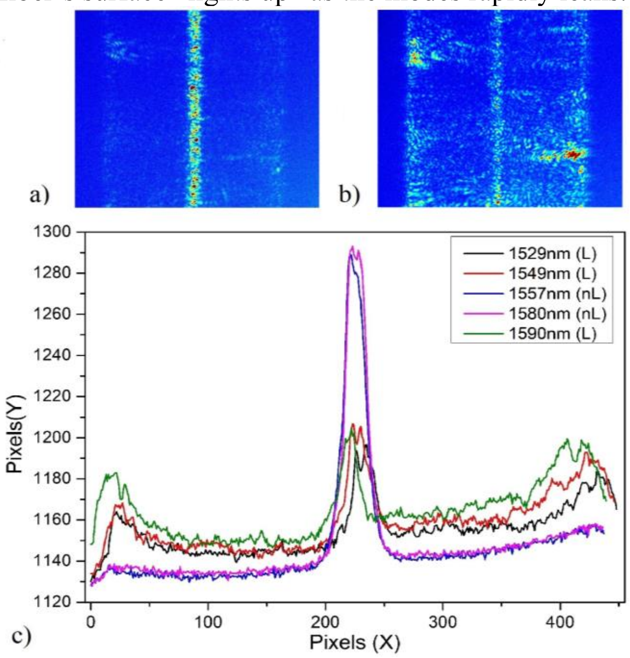


Fig. 8 Infrared camera image of the fiber core when light is launched using a tunable laser. a) Tuned to a wavelength with no LMR (nL). b) Tuned to a wavelength coincident with the LMR (L). c) Pixel intensity analysis showing the light distribution for a), b).

V. CONCLUSION

The excitation of leaky mode resonances that originate from directly coupling light propagating in the fiber core mode was achieved using small angle tilted, high order TFBGs. The nature of the coupling was optimized by using femtosecond laser inscription of tailored tilted gratings (via the use of the plane-by-plane inscription method), selectively and controllably triggering the coupling to backward propagating LMRs. Their nature as leaky modes was confirmed through simulation, spectral measurements (with polarized light and in different refractive index media) and observation of their radiation profile. In this work, the devices were characterized for sensing applications, demonstrating flexibility and robustness, allowing for the measurement of their response to numerous parameters with the same demodulation equipment. In particular, it was shown that the strain sensitivity of an LMR is negative, reaching a value of –2 pm/με, a close to twofold increase in absolute value compared to standard gratings. Differential strain measurements showed a factor-of-three improvement in strain response. By combining the information extracted from the LMR and Bragg resonances; one yields a very well-conditioned system for temperature and strain discrimination. Moreover, the LMR polarization-selective behavior leads to torsion sensing and can find relevance in other applications; beyond the sensing field, such as optical filtering and wavelength tuning.

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