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LASERS
AND THEIR APPLICATIONS

Dynamics of a Wave Packet of Whispering-Gallery-Mode Type in an Optical Waveguide in the Presence of a Traveling Refractive-Index Wave

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Abstract—It is demonstrated that strong ultrafast modulation of a tunneling wave packet of the whispering-gallery-mode type can be implemented in a cylindrical optical waveguide exhibiting spatiotemporal inhomogeneity induced by a travelling refractive-index wave. The corresponding optical waveguides can be used for efficient generation of picosecond and subpicosecond laser pulses.

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INTRODUCTION

As a rule, a wave propagating along the equator of a microsphere and experiencing multiple reflections from the spherical glass–air interface is referred to as a "wave packet (WP) of whispering-gallery-mode (WGM) type" [1–4]. Such waves can be driven also in barrel-shaped sections of an optical fiber [5, 6], as well as in cylindrical optical waveguides [7]. In the latter case, they are referred to as "tunneling modes" [8]. A wave packet of this kind propagates inside a cylindrical glass waveguide along a helix characterized by a constant pitch length. Among specific features of the tunneling waves of the WGM type, of prime importance is the fact that longitudinal (along the optical-waveguide axis) group velocity of such a WP can assume arbitrary values lower than the speed of light in vacuum [9–12].

In this regard, we believe that it will be of interest to analyze the dynamics of a wave packet of tunneling-wave type in optical waveguides in the presence of a travelling refractive-index wave (TRIW) synchronized with propagating radiation [13–16]. The corresponding dynamic variation of refractive index in the waveguide can be achieved by either excitation of an acoustic wave in the waveguide [17, 18] or by arranging refractive-index modulators along the latter [19, 20].

In the present work, we analyze propagation of surface waves slowly tunneling along an optical waveguide. We consider the situation in which a travelling refractive-index wave (TRIW) is driven in the waveguide, and phase velocity of this wave coincides with longitudinal component of the surface-wave group velocity. It is demonstrated that strong modulation and spectral broadening of the WP on a time scale of less than 10 s can take place in the corresponding optical waveguides with varying refractive indices; in so doing, linearity of chirp is preserved.

THE WAVE FIELD OF THE TUNNELING SURFACE WAVE

When light is coupled into a cylindrical waveguide at a certain angle with respect to the generatrix of the cylinder (Fig. 1). In a general case, the surface wave propagates along a helical trajectory [21]. The longitudinal component of the wave vector of such a wave is $k_z = (k^2 - k_r^2)^{1/2}$, where $k = k_0 n(\omega)$; $k_0 = \omega/c$; ω and c are the wave frequency and the speed of light in vacuum, respectively; $n(\omega)$ is the refractive index of the waveguide material; and k_r is the radial component of the wave vector. If the coupling angle of the wave into an optical waveguide is sufficiently close to normal (with respect to the generatrix), propagation of the wave along its axis slows down considerably. Such superslow waves were named the tunneling waves. Similar waves can also be obtained in an inhomogeneous optical waveguide exhibiting a gradually changing diameter. In so doing, relation $k_r a = \text{const}$ must hold, where a is the waveguide diameter, which is a function of longitudinal coordinate z, in the general case. This circumstance offers an additional opportunity of controlling parameter k_r and, as a result, parameter k_z. For instance, the longitudinal component of the group velocity can be reduced (down to zero) in a waveguide with a decreasing diameter due to

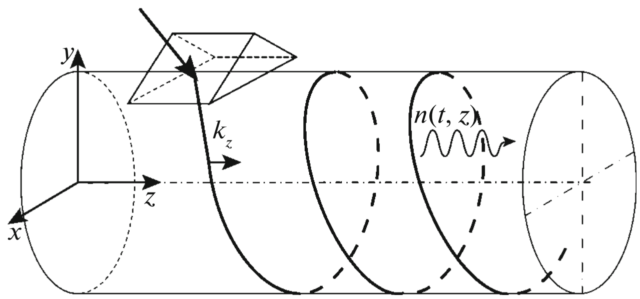

Fig. 1. Prism-coupling of a ray into a cylindrical optical waveguide and its trajectory of WGM type [21].

an increase in the radial component of the wavenumber.

Henceforth, we will analyze the case of a surface wave slowly tunneling (with velocity $V_z \ll c$) along the z axis (therefore, $k_z \ll k \sim k_r$) in an optical waveguide exhibiting a diameter decreasing with length $a(z)$. In this case, the electric field of the wave can be presented in the form

$$E(z,r,t,\varphi) = A(z,t)\Phi(r,z,\varphi)\exp\left(i\omega t - i\int_0^z k_z(z)dz\right), \quad (1)$$

where $A(z,t)$ is the slowly varying amplitude describing longitudinal (along the z axis) propagation of the tunneling-wave field, while function $\Phi(r,z,\varphi)$ determines the radial and azimuthal dependences of the field in the optical waveguide. For a homogeneous cylindrical optical waveguide in the quasi-linear approximation, this function for azimuthal mode of mth order at $r \le a$ can be presented in the form [17]

$$\Phi_m(r,\varphi) = \Phi_0 \,\mathrm{Ai}[\sqrt[3]{2m^2}(1 - r/a) - \lambda_m]\exp(im\varphi), \quad (2)$$

where Ai[...] is the Airy function of the first kind, λ_m is the corresponding root of the Airy function [13, 22], and φ is the angle between the x axis and radius-vector r. We assume that $\Phi_m(r,\varphi) \to 0$ at $r > a$.

The dynamics of the wave-packet temporal envelope in such a waveguide is governed by the equation

$$\frac{\partial A}{\partial z} + V_z^{-1}\frac{\partial A}{\partial t} - iD_z\frac{\partial^2 A}{\partial t^2} = GA, \quad (3)$$

where we introduced the longitudinal (along the z axis) wave velocity, group-velocity dispersion (GVD), and gain coefficient given by

$$V_z(z) \simeq \gamma\frac{\partial\omega}{\partial k}, \quad D_z(z) \simeq \frac{1}{\gamma}\frac{\partial^2 k}{\partial\omega^2}, \quad G(z) \simeq \frac{1}{\gamma}(g - \beta''). \quad (4)$$

Here, g and β'' are the gain coefficient of the material and the damping factor of the waveguide, respectively, and $\gamma(z) = (1 - k_r^2/k^2)^{1/2}$ is the parameter of longitudinal slowdown of the tunneling wave.

Note that the potentially large surface area of the waveguide makes side pumping a convenient way of creating population inversion (whereby high-power pump sources can be used). This method affords achieving population inversion in a large volume of the active (gain) medium and, as a result, obtaining high effective gain $G(z)$ per unit length of the optical waveguide.

EQUATIONS GOVERNING INTERACTION OF A TUNNELING PULSE WITH TRIW

Suppose that a TRIW in which the refractive index of the medium changes according to the relation

$$n(t,z) = n_0[1 - b\cos(\Omega t - qz)] \quad (5)$$

is propagating in an optical waveguide. Here, Ω is the modulation frequency, $q = 2\pi/\Lambda$ is the wavenumber, Λ is the period, $V_a = \Omega/q$ is the TRIW phase velocity, $b = \Delta n/n_0$ is the modulation amplitude, and Δn is the amplitude of refractive-index variation. If an acoustic wave is driven in the waveguide and its phase velocity is $V_a \approx 6\times 10^3$ m/s, parameter of longitudinal slowdown must be on the order of $\gamma \approx V_a/(c/n)$ for acoustic and surface waves to be synchronized. For synchronization of the tunneling WP and the TRIW in a quartz waveguide with standard value of $n \approx 1.5$, it is necessary to have $\gamma \approx 3\times 10^{-5}$. In so doing, the depth of refractive-index modulation can be as high as $|b| = 3\times 10^{-4}$ [22].

Let us analyze the case of co-propagation of a TRIW and a frequency-modulated (FM) Gaussian pulse with the following initial parameters [23, 24]:

$$A(t, z = 0) = A_0\exp(-(\tau_0^{-2} + iC_0)t^2/2), \quad (6)$$

where $A_0 = \sqrt{P_0}$, P_0 is the power of radiation coupled into the optical waveguide, $\tau_0 = \tau_i(0)$ is the initial pulse duration, and C_0 is the rate of FM (chirp) of the pulse. At $V_z \approx V_a$, strong resonance interaction takes place between the tunneling wave and the TRIW. In this case, the equation describing the dynamics of the slowly varying amplitude of longitudinal distribution of the tunneling-wave field, taking into account its interaction with the TRIW, can be expressed in the following form in the retarded frame of reference:

$$\frac{\partial A}{\partial z} - iD_z\frac{\partial^2 A}{\partial\tau^2} - GA = ikb\gamma^{-1}\cos[\Omega(\tau - \delta\tau)]A, \quad (7)$$

where parameter $\delta\tau = (V_a^{-1} - V_z^{-1})z$ characterizes the time offset related to the difference of the pulse group velocity and the TRIW velocity. For pulses characterized by duration $\tau_i|\Omega| \ll 1$ and sufficiently small time offset $|\delta\tau| \approx (10^{-10}–10^{-11})$ s, a power series expansion of the form

$$\cos[\Omega(\tau - \delta\tau)] \approx 1 - \Omega^2(\tau - \delta\tau)^2/2 \qquad (8)$$

can be used in the right-hand side of (7). Equation (7) can thus be recast in the form

$$\frac{\partial A}{\partial z} - iD_z\frac{\partial^2 A}{\partial \tau^2} - GA = i(S_1 + S_2\tau + S_3\tau^2)A, \qquad (9)$$

where functions $S_j(z)$ are defined by the following expressions:

$$S_1 = bk\gamma^{-1}(1 - \Omega^2\delta\tau^2/2), \quad S_2 = bk\gamma^{-1}\Omega^2\delta\tau,$$
$$S_3 = -bk\gamma^{-1}\Omega^2/2.$$

Note that complete synchronization of the waves takes place at $S_2 \to 0$. This condition is achieved by choosing the right coupling angle of radiation into the optical waveguide (or the right value of parameter γ).

FORMATION OF LOCALIZED WAVE PACKETS

Let us analyze in detail the dynamics of initially transform-limited (i.e., $C_0 = 0$) pulses of WGM type in the discussed cylindrical optical waveguides. Let us recast Eq. (9) in the form

$$\frac{\partial B}{\partial z} - iD_z\frac{\partial^2 B}{\partial \tau_n^2} = iS_3\tau_n^2 B, \qquad (10)$$

where we introduced amplitude

$$B = A\exp\left(-i\int_0^z (S_1 - S_2^2/4S_3 - iG)dz\right)$$

and time in the parabolic potential well $\tau_n = \tau + (S_2/2S_3)$. The obtained equation is well known as an equation describing a harmonic oscillator [25].

The corresponding equation at $S_3/D_z < 0$ and $D_z < 0$ has soliton-like solutions similar to the functions of a harmonic oscillator [13, 25]:

$$B(z,\tau_n) = B_0\exp\left(-\frac{\tau_n^2}{2\tau_i^2}\right)H_l(\tau_n/\tau_i)$$
$$\times\exp\left(i\int_0^z \sqrt{-D_z S_3(z)/2}\,dz\right), \qquad (11)$$

where $H_l(\tau)$ is the Chebyshev–Hermite polynomial of lth order that is defined by the relation

$$H_l(\Lambda) = \frac{(-1)^l}{\sqrt{\sqrt{\pi}2^l l!}}\exp(\Lambda^2)\frac{d^l}{d\Lambda^l}(\exp(-\Lambda^2)) \qquad (12)$$

and $\Lambda = \tau_n/\tau_i$. Expression (11) represents a soliton-like pulse propagating along the z axis that has duration $\tau_i \simeq (-D_z/2S_3)^{1/4}$ and energy

$$W(z) = W_0\exp\left[2\int_0^z G(z)dz\right], \qquad (13)$$

where W_0 is the energy of the WP coupled into the waveguide. Note that, in the case under consideration, the soliton-like pulse can be obtained also in a medium with normal dispersion; i.e., at $D_z > 0$. The corresponding regime is realized at $S_3 < 0$ (at $b > 0$), which corresponds to synchronization of the WP with a minimum in dependence $n(t,z)$ in the TRIW. This circumstance makes the WP of this kind more stable with respect to the influence of modulation instability [23, 24].

SUPERSTRONG MODULATION OF THE SURFACE WAVE PACKET

The regime of WP dynamics in which $S_3/D_z > 0$ is of considerable practical interest. Let us analyze the dynamics of an initially frequency-modulated WP with nonzero initial chirp C_0. In this case, the amplitude of the WP envelope can be presented in the form

$$A(z) = B(z)\exp[i(\xi(z)\tau + \alpha(z)\tau^2)], \qquad (14)$$

where ξ represents variation of speed of the envelope maximum in the field of the TRIW caused by the mismatch between the group velocity of the frequency-modulated wave and the TRIW velocity, while α is the rate of the TRIW-induced frequency modulation. Let us substitute expression (14) into (7), which yields the following set of equations governing introduced parameters:

$$\frac{\partial B}{\partial z} + 2D_z(\xi + 2\alpha\tau)\frac{\partial B}{\partial \tau} - iD_z\frac{\partial^2 B}{\partial \tau^2}$$
$$= i(S_1 - \xi^2 D_z + 2i\alpha D_z)B, \qquad (15a)$$

$$\frac{\partial \xi}{\partial z} + 4\alpha D_z\xi = S_2, \qquad (15b)$$

$$\frac{\partial \alpha}{\partial z} + 4D_z\alpha^2 = S_3. \qquad (15c)$$

By using substitution

$$B = \bar{B}\exp\left[i\int_0^z\left(S_1 - \xi^2 D_z + 2i\alpha D_z\right)dz\right], \qquad (16)$$

Eq. (15a) can be recast in the form

$$\frac{\partial \bar{B}}{\partial z} - iD_z(z)f^2(z)\frac{\partial^2 \bar{B}}{\partial \tau'^2} = 0, \qquad (17)$$

where we introduced parameter

$$f(z) = \exp\left(-4\int_0^z D_z(z)\alpha(z)dz\right) \quad (18)$$

and normalized retarded time

$$\tau'(z) \equiv f(z)\tau - 2\int_0^z f(z)D_z(z)\xi(z)dz. \quad (19)$$

A solution to Eq. (17) with initial condition (6) can be presented in the form

$$\bar{B}(\tau', z) = A_0 F^{-1/2}(z)\exp\left[-\frac{(\tau_0^{-2} + iC_0)\tau'^2}{2F(z)}\right], \quad (20)$$

where we introduced the following notations:

$$F(z) = 1 - 2g(z)(C_0 - i\tau_0^{-2}), \quad g(z) = \int_0^z D_z(z)f^2(z)dz.$$

Assuming that $\xi = 0$ and $S_2 = 0$ (when group velocity of the WP and the TRIW velocity are equal and, as a result, $\delta\tau = 0$), we obtain an expression governing the duration of the phase-modulated pulse,

$$\tau_i(z) = \frac{\tau_0}{f(z)}\sqrt{(1 - 2C_0 g(z))^2 + \frac{4g^2(z)}{\tau_0^4}}, \quad (21)$$

and its real chirp

$$C_{ef}(z) = \bar{C}(z) - 2\alpha(z), \quad (22)$$

where

$$\bar{C}(z) = \frac{C_0 - 2(C_0^2 + \tau_0^{-4})g(z)}{(1 - 2C_0 g(z))^2 + 4\tau_0^{-4}g^2(z)}. \quad (23)$$

The rate of TRIW-induced frequency modulation $\alpha(z)$ in (22) can be found from solution to Eq. (15c). For $D_z S_3 > 0$, we have

$$\alpha = \pm\sqrt{\frac{S_3}{4D_z}}\tanh(2\sqrt{S_3 D_z}z), \quad (24)$$

where the plus sign corresponds to $S_3 > 0$ and synchronization of the pulse with refractive-index maximum in the TRIW in the case of normal waveguide-material dispersion ($D_z > 0$), while the minus sign corresponds to $S_3 < 0$ and phase matching of the pulse with the refractive-index minimum in the TRIW in the case of anomalous waveguide-material dispersion ($D_z < 0$).

The obtained solutions show that ultrafast modulation of a slow WP is possible when it interacts with TRIW. In so doing, the linearity of the chirp and large spectral width of the WP are preserved. In turn, this allows achieving further compression of the pulse. In the case of positive chirp, compression can be accomplished by means of diffraction gratings. If, however, the pulse has acquired a negative chirp, a conventional optical waveguide with normal material dispersion can be used for further pulse compression. In both cases, the pulse duration after compression (when pulse becomes transform limited with $C_{ef} \to 0$) is determined by the relation [15, 23]

$$\tau_{min} = \tau_i(z)/\sqrt{1 + C_{ef}^2(z)\tau_i^4(z)} \cong 1/2|\alpha(z)|\tau_i^2(z). \quad (25)$$

It follows from the above relations that, for initial width $\tau_0 = 10^{-11}$ s of a transform-limited ($C_0 = 0$ and $\alpha_0 = 0$) pulse coupled into an optical waveguide-modulator, TRIW modulation depth $bk \sim \pm 2\times 10^2$ m^{-1}, modulation frequency $\Omega \sim 3\times 10^9$ s^{-1}, $\gamma \sim 10^{-5}$, and effective GVD $|D_z| = (10^{-21}$–$10^{-22})$ s^2/m, the rate of frequency modulation on length $z \sim 1$ cm of a cylindrical optical waveguide-modulator will be $|C_{ef}| \approx 10^{24}$ s^{-2}. In so doing, the pulse width does not change substantially. As a result, after propagation through a dispersive element performing temporal compression, the pulse can become shorter by a factor of 100 down to $\tau_i = 10^{-13}$ s.

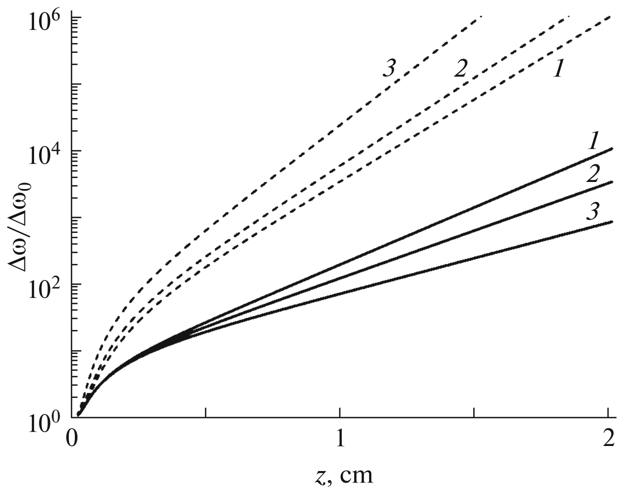

Fig. 2. Normalized spectral width as a function of waveguide length at $b = -3\times 10^{-4}$ and $D_z = (0.5, 0.9, 1.3) \times 10^{-21}$ s^2/m (solid lines *1–3*, respectively); at $D_z = 1 \times 10^{-21}$ s^2/m and $b = (-2, -2.3, -3.2) \times 10^{-4}$ (dashed lines *1–3*, respectively).

Since pulse width is related to its spectral width by relation $\tau_{min} \simeq 1/\Delta\omega(z)$, the spectral width of the corresponding WP is given by

$$\Delta\omega(z) = \sqrt{\tau_i^{-2}(z) + C_{ef}^2(z)\tau_i^2(z)}. \quad (26)$$

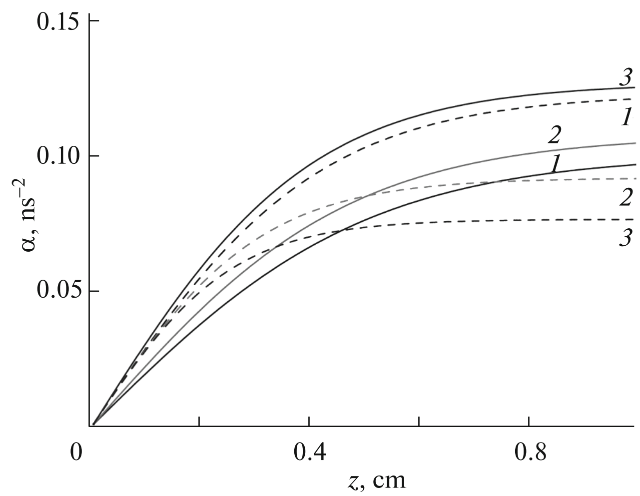

Fig. 3. The dependence of the TRIW-induced chirp on optical-waveguide length at $b = -3\times10^{-4}$ and D_z = (0.5, 0.9, 1.3) × 10^{-21} s^2/m (solid lines *1–3*, respectively); at D_z = 1 × 10^{-21} s^2/m and b = (−2, −2.3, −3.2) × 10^{-4} (dashed lines *1–3*, respectively).

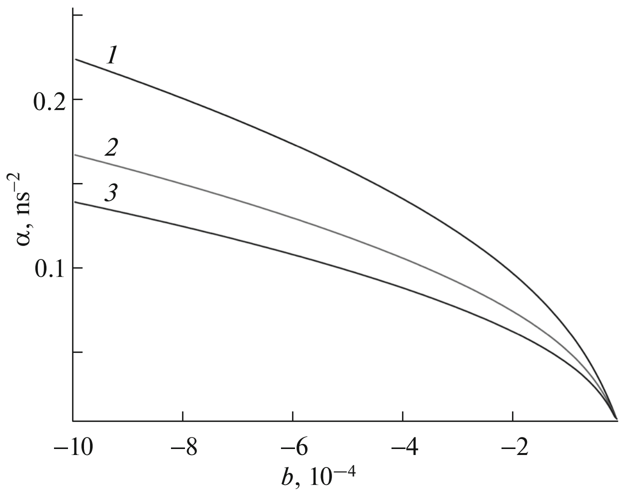

Fig. 4. Pulse chirp as a function of modulation depth b at z = 1 cm for D_z = (0.5, 0.9, 1.3) × 10^{-21} s^2/m (curves *1–3*, respectively).

For considered values of parameters, with high degree of accuracy, $|\bar{C}(z)| \ll |\alpha(z)|$. Therefore, expression (26) can be recast in the form

$$\Delta\omega = \sqrt{\tau_i^{-2} + 4\alpha^2\tau_i^2}. \quad (27)$$

Normalized spectral width of the pulse Δω as a function of path length in the waveguide z is presented in Fig. 2b for the following parameters of the waveguide, WP, and TRIW: $\tau_0 = 10^{-11}$ s, $k = 6\times10^6$ m^{-1}, $\gamma = 3\times10^{-5}$, $\Omega = 10^9$ s^{-1}, $\Delta\omega_0 = 10^{11}$ s^{-1}. Solid lines correspond to modulation depth $b = -3\times10^{-4}$ and dispersion D_z = (0.5, 0.9, 1.3) × 10^{-21} s^2/m (curves *1–3*); dashed lines correspond to $D_z = 10^{-21}$ s^2/m and b = (−2, −2.3, −3.2) × 10^{-4} (curves *1–3*). It can be seen that, for chosen values of parameters, considerable spectral broadening (by a factor of 100 and more) can be achieved at waveguide lengths shorter than 1 cm.

Figure 3 depicts TRIW-induced chirp as a function of waveguide length for the same values of parameters as in Fig. 2. It follows from Fig. 3 that chirp increases linearly as a function of travelled distance before reaching saturation.

The dependence of pulse chirp on modulation depth b at z = 1 cm for the same values of parameters as in the previous diagrams and corresponding to dispersion D_z = (0.5, 0.9, 1.3) × 10^{-21} s^2/m is illustrated in Fig. 4 (curves *1–3*). It follows from Fig. 4 that chirp increases with increasing modulation depth b.

If the pulse is initially synchronized with the refractive-index wave minimum (when $b > 0$), pulse compression must be performed in a medium exhibiting normal dispersion. In this case, an ordinary optical waveguide with normal dispersion can be used as a pulse compressor. However, if the pulse is synchronized with the TRIW maximum ($b < 0$), pulse compression must be performed in a medium exhibiting an anomalous dispersion. In this case, if the peak power of the pulse is lower than 100 W, an ordinary optical fiber exhibiting an anomalous dispersion can be used as a compressor. If the peak power is much higher than 1 kW (which may be of special interest), it would be preferable to use a pair of diffraction gratings to mitigate the nonlinear effects.

CONCLUSIONS

The analysis conducted in the present work shows that optical waveguides in the presence of TRIW propagating at the same velocity as the pulse coupled into the waveguide and having low longitudinal velocity (much lower than the speed of light in vacuum) can be used for strong frequency modulation of the pulse on a short distance (much shorter than 10 cm) of the waveguide modulator. In this case, considerable (by one to three orders of magnitude, up to $\Delta\omega \cong 10^{14}$ s^{-1}) spectral broadening of the pulse can be achieved already at a waveguide distance this short, while preserving linearity of the chirp (i.e., with quadratic temporal dependence of the pulse phase). The later circumstance, in turn, enables strong subsequent compression (temporal compression) of the pulse (by one to three orders of magnitude) down to subpicosecond and femtosecond pulse duration in the optical spectral range. On the other hand, corresponding waveguide modulators (both in fiber-optic and planar geometries) can be used for spectral compression of initially frequency-modulated broadband pulses.

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