Scattering amplitudes via multi-particle higher-spin charges

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Motivation

Why conserved multi-particle higher-spin (HS) charges $Q$ (of free classical HS fields) can be related to scattering amplitudes $A$ (of interacting quantum fields)? Both $Q$ and $A$ are:

- constructed from classical on-shell data,
- non-local objects, i.e. $Q \sim \int_{\text{surface}} J$, $A \sim \exp i \int_{M_4} \text{inkowski} \mathcal{L}$.

- $\infty$-many $Q$’s (functional ambiguity) $\Rightarrow$ probably large enough space to embed expressions for scattering amplitudes of QFT’s.

Paradox alert!

How can it happen that a single free classical HS theory can produce scattering amplitudes of various interacting QFT’s?

Resolution: the approach in question does not reconstruct amplitudes from non-linear dynamics, but rather provides us with a large kinematical space where (probably all) amplitudes of (probably all) QFT’s can be embedded.

Example of MHV amplitudes

We aim at reproducing Parke-Taylor formula for color-ordered tree MHV amplitudes for $n$ gluons with $i^-, \ j^-$.

$$a_{\mu_1, \ldots, \mu_n} \frac{\langle \mu_1 \mu_2 \rangle_4 \cdots \langle \mu_{n-1} \mu_n \rangle_4 \delta^4 \left( \sum_{i=1}^{n \ (i \neq a)} \mu_i \tilde{\eta}_{i \mu} \right) \cdots \delta^4 \left( \sum_{i=1}^{n \ (i \neq n)} \mu_i \tilde{\eta}_{i \mu} \right)}{\det \tilde{\eta}_{\mu_1 \cdots \mu_n} \left( \mu = \frac{1}{2} \text{det} \eta_{a \mu} \right) \ldots \left( \mu = \frac{1}{2} \text{det} \eta_{n \mu} \right)}$$

where $\mu, \tilde{\mu}_a$ are SL(2, C) momentum spinors such that $\eta_{a \mu} = \mu_\tilde{\mu}_a$ ($\mu^2 = \frac{1}{2} \det \eta_{a \mu} = 0$) and $(\mu, \tilde{\mu}) = e^{\frac{\pi}{2} i} \mu_\tilde{\mu}$.

MHV amplitudes as HS charges

To reproduce a given amplitude there are “free parameters” on the HS side to adjust: $r = m + \bar{m}$ and HS symmetry parameter $\eta$. For 3-particle MHV amplitude we take $r = 3$ with $m = 2, \bar{m} = 1$ and $\eta = \eta^{(3,2)} := \prod_{i=1,2} \text{sign}(\eta_i)$. Then

$$Q^{(2,1)} \left[ \eta^{(12,5)} \right] \propto A^{(1,2,3)}.$$ The result is independent of the integration surface $S \subset \text{Cor}(2,1)$.

Concluding remarks

- Scattering amplitudes are related to HS multi-particle charges of classical free HS theory. We expect that our result extends beyond tree level and QCD gluon amplitudes. This means, in particular, that amplitudes are embedded in the HS multiparticle symmetry.

- Various approaches for amplitude calculations can be expected to be related via a properly extended space $\text{Cor}$ and an on-shell-closed differential form by different choices of integration surface.

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Unfolded HS dynamics

Higher-spin fields are represented by 0-forms $C^i(y, \tilde{y}; x)$ obeying rank-1 unfolded equation

$$\left( \frac{\partial}{\partial y^{\mu a}} + i \frac{\partial^2}{\partial \tilde{y} \partial y^{\mu a}} \right) C^i(y, \tilde{y}; x) = 0,$$

$$\eta_{a \mu} = \frac{1}{2} \det \eta_{a \mu} \left( \mu = \frac{1}{2} \text{det} \eta_{a \mu} \right) \ldots \left( \mu = \frac{1}{2} \text{det} \eta_{n \mu} \right)$$

is a formal power series, then unfolded equation describes on-shell gauge-invariant field strengths of all half-integer spins: spin-0 field $c(x)$, spin-1/2 fields (Weyl fermions) $c_a(x)$, spin-1 field (self-duality components of Maxwell tensor) $c_a(x)$, etc. Mixed components $c_{a_1 \ldots a_n}$ are auxiliary and represent all on-shell derivatives fields. We deal with distributions which do not admit projection to a particular spin, for example plane-wave solutions

$$\chi_{\mu, \tilde{\mu}}(y, \tilde{y}; x) = \exp \left[ i \left( \mu_\tilde{\mu} \tilde{\eta}_{\mu a} - \mu a \tilde{\eta}_{\tilde{\mu} a} \right) \right]$$

with light-like momenta $i \frac{\partial}{\partial \mu \tilde{\mu}} \propto \mu_\tilde{\mu}$.

Multiparticle states $= \text{tensor product}$

Multiparticle states of HS fields are described by their products $C^{(l)}(y_1 \ldots x) \cdot C^{(l)}(y_2 \ldots x)$ which are particular case of rank-$r$ fields satisfying rank-$r$ unfolded equation

$$\left( \frac{\partial}{\partial y^{\mu a}} + i \frac{\partial^2}{\partial \tilde{y} \partial y^{\mu a}} \right) C^r(y, \tilde{y}; x) = 0,$$

where $I, J = 1 \ldots r$. The case $r = 2$ (bilinear fields) corresponds to conserved HS currents, higher ranks $r \geq 3$ appear to correspond to scattering amplitudes. For a partition $r = m + \bar{m}$ half-Fourier transform in variables $y_i^\mu (i = 1 \ldots m)$ and $\tilde{y}_j^{\tilde{\mu}} (j = m + 1 \ldots r)$ gives a first-order PDE

$$\left( \frac{\partial}{\partial y^{\mu a}} - \lambda_{\mu a} \frac{\partial}{\partial \tilde{y}^{\tilde{\mu} b}} - \lambda_{\tilde{\mu} b} \frac{\partial}{\partial \tilde{y}^{\tilde{\mu} b}} \right) g^{\tilde{y} \mu; \tilde{y} \mu} \left( \tilde{y}^\mu, \tilde{y}^{\tilde{\mu}} \right) = 0$$

with characteristics $\lambda_{\mu a} = \tilde{y}^{\tilde{\mu} b} + x^{\mu a}$ and $\tilde{y}^{\tilde{\mu} b} + x^{\mu a} \tilde{y}^{\tilde{\mu} b}$. The solutions of our interest are $\Omega_{\mu \tilde{\mu}}(\tilde{y}; x)$, $\chi_{\mu, \tilde{\mu}}$, where $\chi_{\mu, \tilde{\mu}}(\tilde{y}; x) \equiv \Omega_{\mu \tilde{\mu}}(\tilde{y}; x)$, and $\eta$ is an arbitrary functional (HS-symmetry) parameter.

On-shell-closed differential form

Fields $g^{\tilde{y} \mu; \tilde{y} \mu}(\tilde{y}, \mu; \tilde{y}, \mu; x)$ live in the extended space $\text{Cor}(m, \bar{m}) = M_4 \times \mathbb{R}^{4r}$ and allow us to build an on-shell-closed differential form

$$\Omega_{\mu \tilde{\mu}}^{(m, \bar{m})}[y] = d^{2m} \Lambda d^{2m} \Lambda d^{2m} \left( y + x \Lambda \right) d^{2m} \left( \tilde{y} + x \Lambda \right) \eta \chi_{\mu \tilde{\mu}},$$

such that $d_{\text{Cor}} \Omega_{\mu \tilde{\mu}}^{(m, \bar{m})}[y] = 0$. This gives a charge

$$Q_{\mu \tilde{\mu}}^{(m, \bar{m})}[y] := \int_{S} \mathcal{L}_{\mu \tilde{\mu}}^{(m, \bar{m})}[y]$$

which is conserved in a sense that it does not depend on local variations on integration surface $S \subset \text{Cor}(m, \bar{m})$. In particular, one gets the same answer by integrating equally over both spacetime or spinor variables.