Frequently hypercyclic operators, and related notions
Karl-G. Große-Erdmann

In linear dynamics we study the behaviour of orbits \( \{x, Tx, T^2x, \ldots \} \) of vectors \( x \in X \) under (continuous and linear) operators \( T : X \to X \), where \( X \) is typically a Banach- or Fréchet-space. An operator is called hypercyclic if it admits a dense orbit. In 2004, Bayart and Grivaux introduced the concept of a frequently hypercyclic operator: it admits a vector whose orbit not only meets any non-empty open set \( U \) at least once (and hence infinitely often – like for hypercyclicity) but very often indeed in the sense that \( \text{dens}\{n \geq 0 : T^n x \in U\} > 0 \). We will start by discussing the fundamental properties of frequently hypercyclic operators.

Now, the definition of Bayart and Grivaux involves the family \( \mathcal{A}_{ld} \) of subsets of \( N_0 \) of positive lower density. Recently, researchers have defined the general notion of an \( \mathcal{A} \)-hypercyclic operator by replacing the family \( \mathcal{A}_{ld} \) by an arbitrary family \( \mathcal{A} \) of subsets of \( N_0 \). We will discuss some of the properties of this new notion. The results obtained help to understand why, in certain respects, frequent hypercyclicity behaves markedly differently from (ordinary) hypercyclicity.