Games where you can play optimally with finite memory

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Highlights of Logic, Games and Automata 2019

Games where you can play optimally with finite memory

⚠ Work in progress ⚠

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A sequel to the critically acclaimed blockbuster by Gimbert & Zielonka

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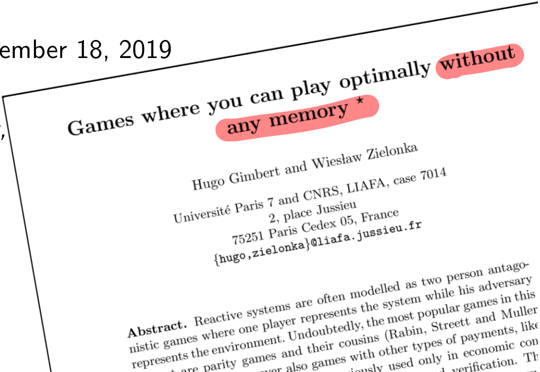

Games where you can play optimally without any memory *

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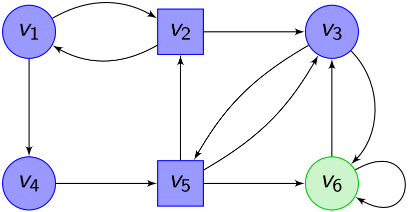
Abstract. Reactive systems are often modelled as two person antago-
nistic games where one player represents the system while his adversary
represents the environment. Undoubtedly, the most popular games in this
... are parity games and their cousins (Rabin, Streett and Muller
...

Two-player turn-based zero-sum games on graphs

We consider *finite* arenas with vertex *colors* in C. Two players: circle ($\mathcal{P}_1$) and square ($\mathcal{P}_2$). Strategies $C^* \times V_i \to V$.



From where can $\mathcal{P}_1$ ensure to reach v_6?
How complex is his strategy?

Memoryless strategies ($V_i \to V$) always suffice for reachability (for both players).

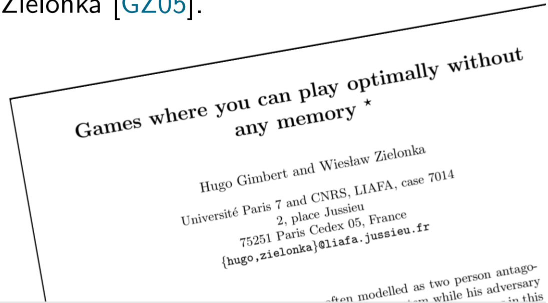
When are memoryless strategies sufficient to play optimally?

Virtually always for **simple** winning conditions!

Examples: reachability, safety, Büchi, parity, mean-payoff, energy, total-payoff, average-energy, etc.

Can we characterize when they are?

Yes, thanks to Gimbert and Zielonka [GZ05].



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often modelled as two person antago-
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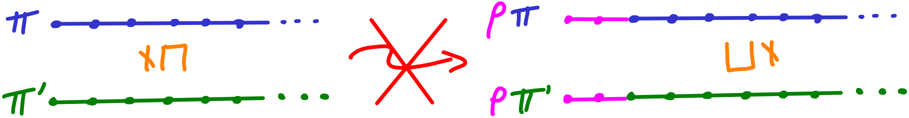
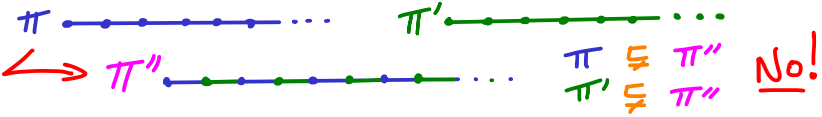
Gimbert and Zielonka's characterization

Memoryless strategies suffice for a *preference relation* $\sqsubseteq$ (and the induced winning conditions) **if and only if**

1. it is **monotone**,
   - ▷ Intuitively, stable under prefix addition.



2. it is **selective**.
   - ▷ Intuitively, stable under cycle mixing.



Example: reachability.

Gimbert and Zielonka's corollary

If $\sqsubseteq$ is such that

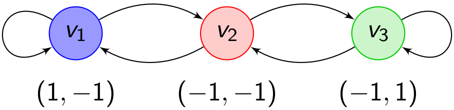
- in all $\mathcal{P}_1$-arenas, $\mathcal{P}_1$ has an optimal memoryless strategy,
- in all $\mathcal{P}_2$-arenas, $\mathcal{P}_2$ has an optimal memoryless strategy (i.e., for $\sqsubseteq^{-1}$),

then both players have optimal memoryless strategies **in all two-player arenas**.

* Extremely useful in practice! *

Going further: finite memory

Memoryless strategies do not always suffice!



Examples:

- Büchi for v_1 *and* v_3 $\rightarrow$ **finite** (1 bit) memory.
- Mean-payoff (average weight per transition) ≥ 0 on all dimensions $\rightarrow$ **infinite** memory!

We need a GZ equivalent for finite memory!

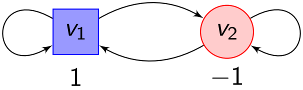
$\rightsquigarrow$ For *combinations*, see [LPR18].

A partial counter-example (lifting corollary)

Let $C \subseteq \mathbb{Z}$ and the winning condition for $\mathcal{P}_1$ be

$$\overline{TP}(\pi) = \infty \quad \vee \quad \exists^{\infty} i \in \mathbb{N}, \sum_{i=0}^{n} c_i = 0$$

Both 1-player variants are finite-memory determined.



↯ No exact equivalent to GZ ↯

But the two-player one is not!
$\Longrightarrow$ $\mathcal{P}_1$ **needs infinite memory to win.**

Hint: non-monotony is a bigger threat in two-player games.
In one-player games, *finite* memory may help.

A new hope

Our goal

GZ-like characterization for finite-memory strategies.

Two tricks:

1. **Monotonicity as hypothesis** (cf. counter-example).
2. From selectivity to $\mathcal{S}$**-selectivity** and **cyclic covers** for arenas.

$\Longrightarrow$ Intuitively, selectivity *modulo a memory skeleton*.

We obtain a natural GZ-equivalent for FM determinacy, including the lifting corollary (1-p. to 2-p.)!

Still some elements to flesh out.

$\Rightarrow$ Preprint writing in progress.

Thank you! Any question?

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