Symmetry breaking and high-frequency periodic oscillations in mutually coupled laser diodes

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We investigate the dynamical behavior of two laser diodes coupled through mutual injection of their optical fields when placed face to face with a small separation between them. We report symmetry breaking in periodic solutions at low coupling rates. In addition, we demonstrate that at higher coupling rates both lasers exhibit very fast periodic oscillations. The system is of practical interest, since it constitutes a tunable all-optical source of microwave oscillations. © 2003 Optical Society of America

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Coupled lasers have attracted much attention in recent years because of their potential applications in areas as different as high-power emission by diode-laser arrays,[1] quantum-noise reduction,[2] and chaotic communications.[3] But coupling can also be a source of dynamical instabilities, as has been observed in solid-state laser arrays[4] and face-to-face coupled semiconductor lasers.[5–7] In this case the time it takes the light to travel from one laser to the other usually has to be considered. Here we analyze the case of a small separation between the lasers of the order of millimeters as a function of the coupling strength that measures the amount of mutually injected light. We show that at a low coupling rate symmetry breaking in periodic solutions occurs: The optical powers emitted by both lasers are periodically modulated with the same frequency but with different amplitudes, even though the system is perfectly symmetric. In addition, we demonstrate that at a higher coupling rate both lasers exhibit high-frequency periodic oscillations (>10 GHz) with identical amplitudes. High-frequency pulsation has also recently been predicted in a system of evanescently (side-by-side) coupled vertical-cavity surface-emitting lasers.[8] In contrast, in our case coupling is not instantaneous, and in fact our results show that the frequency of the oscillations is closely related to the inverse of the time of flight between the lasers. This system is therefore of practical interest, since it constitutes a tunable all-optical source of stable microwave oscillations.

The system that we consider consists of two laser diodes coupled by mutual injection of their optical fields. The two lasers are assumed to be perfectly symmetric; i.e., their internal and operating parameters are identical. This system can be modeled by the following dimensionless equations[6]:

$$\frac{\mathrm{d}E_{1,2}}{\mathrm{d}s} = (1 + i\alpha)N_{1,2}E_{1,2} + \eta E_{2,1}(s - \theta)\exp(-i\Omega\theta), \tag{1}$$

$$T\frac{\mathrm{d}N_{1,2}}{\mathrm{d}s} = P - N_{1,2} - (1 + 2N_{1,2})|E_{1,2}|^2. \tag{2}$$

$E_{1,2}$ and $N_{1,2}$ are the normalized, slowly varying, complex electric fields and the normalized excess carrier numbers, respectively, in lasers 1 and 2. The dimensionless time s is measured in units of the photon lifetime τ_p, where $s = t/\tau_p$. η is the normalized coupling rate, and θ is the ratio of the time of flight of the light between the lasers to the photon lifetime, where $\theta = \tau/\tau_p$. α is the linewidth enhancement factor, and Ω is the product of the angular frequency of a solitary laser and the photon lifetime. P is the dimensionless pumping current above the solitary laser threshold, and T is the ratio of the carrier lifetime to the photon lifetime. We use typical values for the linewidth enhancement factor and the ratio of the carrier lifetime to the photon lifetime, namely, $\alpha = 4$ and $T = 1710$. The other parameters are $P = 1.155$ and $\theta = 20$. For a photon lifetime $\tau_p = 1.11$ ps the value of θ that we chose corresponds to a time of flight of $\tau = 22.2$ ps or equivalently to an interlaser separation of 6.66 mm.

Bifurcation diagrams for laser 1 [Figs. 1(a) and 1(c)] and laser 2 [Figs. 1(b) and 1(d)] show the extrema of the intensity emitted by laser 1 ($|E_1|^2$) and laser 2 ($|E_2|^2$) as functions of η. Figures 1(c) and 1(d) are enlarged views of Figs. 1(a) and 1(b), respectively. At very low coupling the output of both lasers is stationary. At $\eta = 2.7 \times 10^{-4}$ the system loses its stability through a Hopf bifurcation, and both lasers start to oscillate periodically with a frequency $f = 5.15$ GHz that is very close to the relaxation oscillation frequency $f_R = 5.27$ GHz of the individual lasers in the absence of coupling. From $\eta = 2.7 \times 10^{-4}$ to $\eta = 6.1 \times 10^{-4}$ both lasers oscillate with the same amplitude and frequency but in antiphase. The optical spectra of both lasers are identical, almost symmetric with respect to the optical carrier frequency, and exhibit peaks close to the free-running relaxation oscillation frequency and its harmonics. As an example, Figs. 2(a) and 2(c) display time traces of the two

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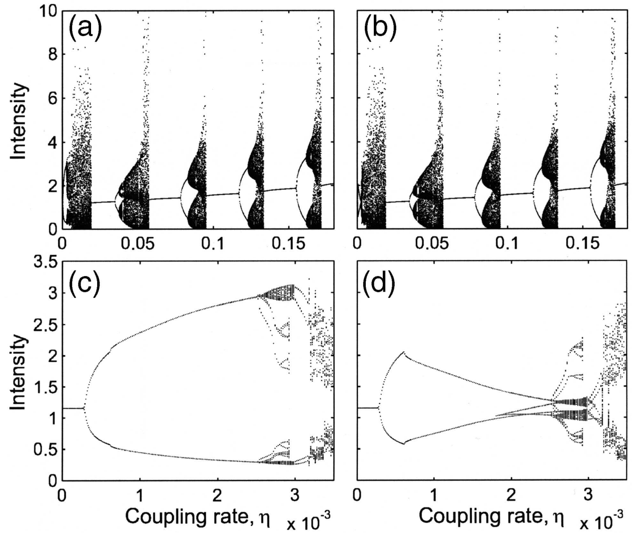

Fig. 1. Bifurcation diagrams of the optical power emitted by (a), (c) laser 1 and (b), (d) laser 2. (c), (d) Enlarged views of (a) and (b), respectively. The coupling rate is the bifurcation parameter.

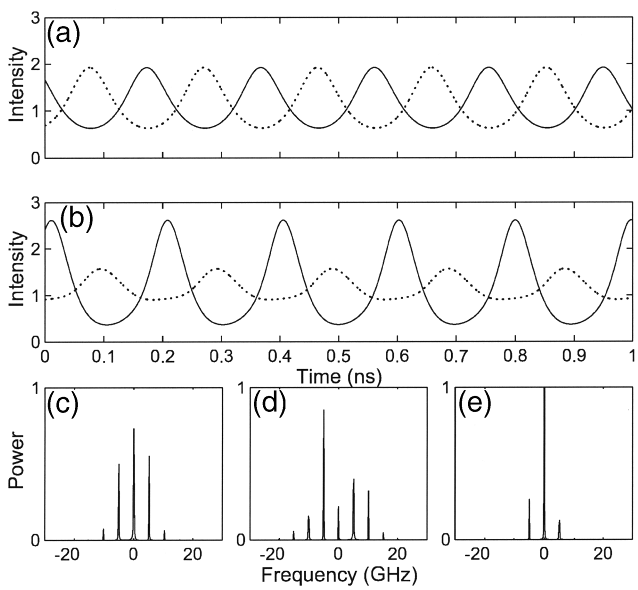

Fig. 2. (a), (b) Time traces of the outputs of laser 1 (solid curve) and laser 2 (dotted curve) for $\eta = 5 \times 10^{-4}$ and $\eta = 1.5 \times 10^{-3}$, respectively. (c) Optical spectrum of laser 1 for $\eta = 5 \times 10^{-4}$. (d), (e) Optical spectra of laser 1 and laser 2, respectively, for $\eta = 1.5 \times 10^{-3}$.

laser outputs and the optical spectrum of laser 1 for $\eta = 5 \times 10^{-4}$. At this point we note that periodic solutions with identical amplitudes in lasers 1 and 2 of the form $E_1(s) = E_2(s - x)$ and $N_1(s) = N_2(s - x)$ are necessarily either in phase with $x = 0$ or in antiphase with $x = \Theta/2$, Θ being the normalized period. We can readily show this by introducing these solutions into Eqs. (1) and (2). In the case of periodic in-phase oscillations (i.e., with $x = 0$) the system behaves as a set of two noncoupled, identical external-cavity laser diodes. However, these solutions are unstable.

The amplitude of the periodic oscillations increases steadily until $\eta = 6.1 \times 10^{-4}$ (Fig. 1). At this point the symmetrical dynamics of the system is destroyed by a symmetry-breaking bifurcation: Although both lasers continue to oscillate at the same frequency, the modulation amplitude is different, and this difference increases with the coupling rate. Figures 2(b), 2(d), and 2(e) show time traces and optical spectra corresponding to asymmetric periodic oscillations for $\eta = 1.5 \times 10^{-3}$. Laser 1 oscillates strongly while laser 2 oscillates weakly, even though both oscillate around the same average value. In addition, the oscillations are no longer in antiphase. The optical spectra are strongly dissimilar: The sidebands are damped in the spectrum of laser 2 [Fig. 2(e)], whereas new sidebands appear in the spectrum of laser 1 [Fig. 2(d)]. We point out that the respective roles of the lasers can be exchanged and depend on the initial conditions. This demonstrates the coexistence of two asymmetric limit cycles in phase space. At $\eta = 2.5 \times 10^{-3}$ a bifurcation to quasi-periodic oscillations with two incommensurable frequencies occurs. It must be noted that periodic solutions coexist with the quasi-periodic solutions in the range $\eta = 2.5 \times 10^{-3}$ to $\eta = 2.9 \times 10^{-3}$ [Figs. 1(c) and 1(d)]. Chaos is observed from $\eta = 3.18 \times 10^{-3}$ to $\eta = 1.9 \times 10^{-2}$. From $\eta = 1.9 \times 10^{-2}$ the system exhibits a cascade of bifurcations with regions where the intensities emitted by the lasers are stationary interspersed with regions of more complex behaviors, such as periodic, quasi-periodic, and chaotic oscillations. Symmetry breaking in periodic solutions is no longer observed, although there is some evidence of symmetry breaking when quasi-periodic solutions are frequency locked. As we show in the following, the solutions in these periodic windows are characterized by a high oscillation frequency (>10 GHz) and are intrinsically different from those observed in the first periodic window.

Figures 3(a) and 3(b) were computed for $\eta = 0.16$ and display a representative example of high-frequency periodic oscillations. The two lasers oscillate exactly in antiphase with the same amplitude and a frequency $f = 17.2$ GHz. The optical spectra are identical and are characterized by the existence of two main peaks, the harmonics being barely visible [see Fig. 3(b), which displays the spectrum of laser 1 only]. This kind of spectrum suggests that the high-frequency periodic oscillations result from a beating between two pairs of single-frequency rotating wave solutions of the form $E_{1a,b}(s) = A_{1a,b} \exp[i(\Delta_{a,b} - \Omega)s]$ for laser 1

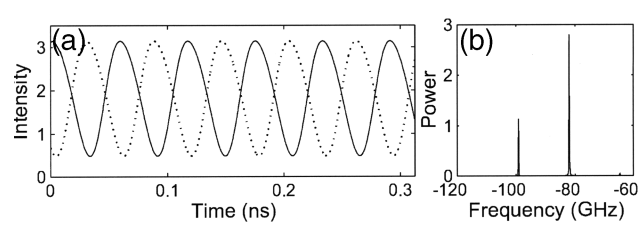

Fig. 3. (a) Time traces of the output of laser 1 (solid curve) and laser 2 (dotted curve). (b) Optical spectrum of laser 1. The coupling strength is $\eta = 0.16$.

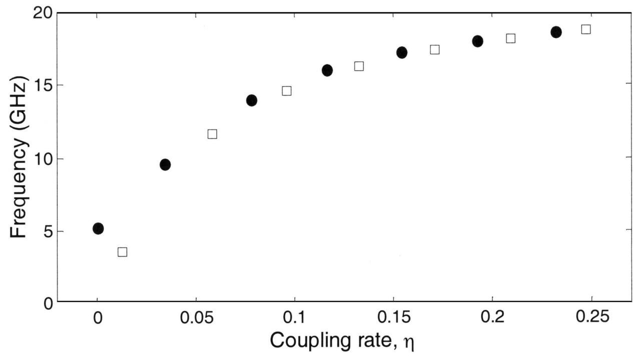

Fig. 4. Frequency of periodic solutions versus coupling rate as obtained numerically (circles) from Eqs. (1) and (2) and analytically (squares) from Eqs. (5)–(7).

and $E_{2a,b}(s) = A_{2a,b} \exp[i(\Delta_{a,b} - \Omega)s]$ for laser 2 and that they can be approximated by analytical solutions of the form

$$E_j(s) = A_{ja} \exp[i(\Delta_a - \Omega)s] + A_{jb} \exp[i(\Delta_b - \Omega)s], \tag{3}$$

with $j = 1, 2$. The corresponding intensities are

$$|E_j(s)|^2 = |A_{ja}|^2 + |A_{jb}|^2 + 2|A_{ja}|\,|A_{jb}|\cos(\omega s + \phi_j), \tag{4}$$

where ϕ_1 and ϕ_2 are constant phases and ω is the normalized beating angular frequency, defined as $\omega = |\Delta_a - \Delta_b|$. The corresponding oscillation frequency is $f = \omega/2\pi\tau_p$. The lasers oscillate in antiphase; thus $\phi_2 = \phi_1 \pm \pi$. By following a procedure similar to that in Ref. 9, it is possible to show that solutions of the form of Eq. (3) are exact solutions of Eqs. (1) and (2) when the ratio of the carrier lifetime to the photon lifetime is large (i.e., for $T \to \infty$) and for particular values of the coupling rate. These values of η and the corresponding normalized angular frequencies can be found by substituting Eq. (3) into Eqs. (1) and (2) and taking into account that N_1 and N_2 are constant in the limit $T \to \infty$. They read as, respectively,

$$\eta_k = [-\Delta_a\theta + (2k + 1)\pi/2]/[\theta \sin(\Delta_a\theta)], \tag{5}$$

$$\omega_k = [2\Delta_a\theta + (2k + 1)\pi]/\theta, \tag{6}$$

with $k = 0, 1, 2, 3 \ldots$. The phases $\Delta_a\theta$ are obtained by solution of the following transcendental equation:

$$\alpha \cot(\Delta_a\theta)\left[\Delta_a\theta + (2k + 1)\frac{\pi}{2}\right] = -\Omega\theta - (2k + 1)\frac{\pi}{2}\,. \tag{7}$$

The frequency ω_k in Eq. (6) is an approximation of the normalized angular frequency of oscillation in the $(k + 1)$th periodic window. An analytical study of Eq. (7) shows that the phase $\Delta_a\theta$ tends to $-k\pi$ as k (i.e., the coupling rate) increases. As a result, we observe from Eq. (6) that the upper limit of the oscillation frequency is $f_\infty = 1/2\tau$. In the numerical case under study this frequency is $f = 22.5$ GHz. Figure 4 shows the dependence of the oscillation frequency on the coupling rate, as obtained numerically from Eqs. (1) and (2) at the successive Hopf bifurcation points and analytically from Eqs. (5)–(7). The figure shows that the agreement between approximate and exact solutions improves as the coupling rate increases.

In summary, we have investigated the dynamics of two mutually coupled laser diodes. We have reported symmetry breaking in periodic solutions at a low coupling rate, and we have demonstrated that high-frequency periodic oscillations can be generated at a higher coupling rate. The frequency of these periodic oscillations is proportional to the inverse of the time of flight between the lasers. The system under study is therefore of practical interest, since it constitutes a tunable all-optical source of microwave oscillations. Finally, it is interesting to note that similar high-frequency periodic solutions have been observed in laser diodes subjected to optical feedback (Refs. 9 and 10 and references therein). Our present results show that the beating between two pairs of single-frequency rotating wave solutions does not require optical feedback to occur.

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