Fano Resonances in Hyperbolic metamaterial-based cavities

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Overview

- Introduction to hyperbolic metamaterials (HMMs)
- Some properties
- Hyperbolic cavities with Fano resonances
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Metamaterials: « material engineered to have a property that is not found in nature »

Building blocks: subwavelength « meta-atoms »

Optical properties from design rather than base materials

Applications: negative refractive index, invisibility cloak, epsilon-near-zero metamaterials, epsilon-near-pole metamaterials, hyperlens, ...
Hyperbolic metamaterial: anisotropic media

\[ \begin{bmatrix} \varepsilon_\parallel & 0 & 0 \\ 0 & \varepsilon_\parallel & 0 \\ 0 & 0 & \varepsilon_\perp \end{bmatrix} \]

Standard effective medium theory (Bruggeman):

\[ \varepsilon_\parallel = f \varepsilon_m + (1 - f) \varepsilon_d \]

\[ \varepsilon_\perp = \frac{\varepsilon_m \varepsilon_d}{\varepsilon_m (1 - f) + \varepsilon_d f} \]

\[ \frac{k_\parallel^2}{\varepsilon_\parallel} + \frac{k_\perp^2}{\varepsilon_\perp} = k_0^2 \]

TM or p-polarization

Metal fill factor
Example with Ag and TiO$_2$

\[ f = \frac{1}{3} \]
\[ d_{\text{Ag}} = 10 \text{ nm} \]
\[ d_{\text{TiO}_2} = 20 \text{ nm} \]

\[ \lambda = 500 \text{ nm} - \text{elliptic} \]
Example with Ag and TiO$_2$

$\frac{k_{\parallel}^2}{\varepsilon_{\perp}} + \frac{k_{\perp}^2}{\varepsilon_{\parallel}} = \frac{\omega^2}{c^2}$

Hyperbolic isofrequency curve!

$\varepsilon_{\parallel} \cdot \varepsilon_{\perp} < 0$ possible

$\lambda = 500$ nm - elliptic

$f = \frac{1}{3}$

$d_{\text{Ag}} = 10$ nm

$d_{\text{TiO}_2} = 20$ nm
Example with Ag and TiO$_2$

\[ \varepsilon_\parallel \varepsilon_\perp < 0 \text{ possible} \]

\[ \frac{k_\parallel^2}{\varepsilon_\perp} + \frac{k_\perp^2}{\varepsilon_\parallel} = \frac{\omega^2}{c^2} \]

Hyperbolic isofrequency curve!

\[ \lambda = 500 \text{ nm} - \text{elliptic} \]

\[ \lambda = 700 \text{ nm} - \text{hyperbolic} \]

\[ f = 1/3 \]

\[ d_{\text{Ag}} = 10 \text{ nm} \]

\[ d_{\text{TiO}_2} = 20 \text{ nm} \]

\[ \lambda = 500 \text{ nm} - \text{elliptic} \]

\[ \lambda = 700 \text{ nm} - \text{hyperbolic} \]

\[ \begin{align*}
  \frac{k_\parallel}{\varepsilon_\perp} + \frac{k_\perp}{\varepsilon_\parallel} &= \frac{\omega^2}{c^2} \\
  \lambda &= \frac{\pi}{k_\parallel + k_\perp} \\
  \omega &= \frac{2\pi}{\lambda} \sqrt{\varepsilon_\parallel k_\parallel + \varepsilon_\perp k_\perp} \\
  f &= \frac{1}{3} \\
  d_{\text{Ag}} &= 10 \text{ nm} \\
  d_{\text{TiO}_2} &= 20 \text{ nm} \\
\end{align*} \]
Group velocity

Preferred direction of propagation along a cone!

B. Wood, J. B. Pendry, and D. P. Tsai
Limits of EMT

\[
\frac{k_{\parallel}^2}{\varepsilon_{\perp}} + \frac{k_{\perp}^2}{\varepsilon_{\parallel}} = \frac{\omega^2}{c^2}
\]
Limits of EMT

\[ \frac{k_{\parallel}^2}{\varepsilon_{\perp}} + \frac{k_{\perp}^2}{\varepsilon_{\parallel}} = \frac{\omega^2}{c^2} \]

Origin of hyperbolic properties: plasmonic
→ Nonlocality
Limits of effective medium theory

\[
\cos (k_y D) = \frac{(\kappa_d \varepsilon_m + \kappa_m \varepsilon_d)^2}{4\kappa_d \kappa_m \varepsilon_d \varepsilon_m} \cosh (\kappa_d d_d + \kappa_m d_m) - \frac{(\kappa_d \varepsilon_m - \kappa_m \varepsilon_d)^2}{4\kappa_d \kappa_m \varepsilon_d \varepsilon_m} \cosh (\kappa_d d_d - \kappa_m d_m)
\]

\[
\kappa_{m,d} = \sqrt{k_x^2 - \varepsilon_{m,d} k_0^2}
\]
Limits of effective medium theory

\[ \cos (k_y D) = \frac{(\kappa_d \varepsilon_m + \kappa_m \varepsilon_d)^2}{4 \kappa_d \kappa_m \varepsilon_d \varepsilon_m} \cosh (\kappa_d d_d + \kappa_m d_m) - \frac{(\kappa_d \varepsilon_m - \kappa_m \varepsilon_d)^2}{4 \kappa_d \kappa_m \varepsilon_d \varepsilon_m} \cosh (\kappa_d d_d - \kappa_m d_m) \]

\[ \kappa_{m,d} = \sqrt{k_x^2 - \varepsilon_{m,d} k_0^2} \]

**Standard effective medium approach (EMT) not valid in many cases**
Fano resonances

Slowly varying background

Narrow resonances
Fano resonances

Slowly varying background + Narrow resonances = Asymmetric Fano resonances

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High-k propagating waves

High-k waves can propagate inside HMM → Possibility to overcome diffraction limit

Application: hyperlens


Extremely high PDOS

Nonresonant phenomena $\rightarrow$ Broadband extremely high PDOS
Spontaneous emission engineering possible


Negative refraction

Isotropic

$k_i$

HMM

$k_i$

$k_i$

$k_i$
Negative refraction
Negative refraction

Isotropic

HMM

$k_i$

$\mathbf{v}_g$

$\mathbf{k}_r$


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Reflection and transmission in slanted cavities
Reflection and transmission in slanted cavities

Right and left: simple multilayer HMM

Centre: « asymmetric hyperbolic metamaterial » (tilted optical axis)
1\textsuperscript{st} model: EMT

\[
\frac{k_\parallel^2}{\varepsilon_\perp} + \frac{k_\perp^2}{\varepsilon_\parallel} = k_0^2
\]

- Propagative mode
- Evanescent mode
EMT of the asymmetric HMM

\[
\bar{\varepsilon} = \mathcal{R}(\theta) \bar{\varepsilon}' \mathcal{R}(\theta)^T = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{pmatrix}
\]

\[
k_x^{(1,2)} = \frac{k_y \varepsilon_{xy} \pm \sqrt{(\varepsilon_{xy}^2 - \varepsilon_{xx} \varepsilon_{yy})(k_y^2 - k_0^2 \varepsilon_{xx})}}{\varepsilon_{xx}}
\]
EMT of the asymmetric HMM

\[ \bar{\varepsilon} = R(\theta) \bar{\varepsilon}' R(\theta)^T = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{pmatrix} \]

\[ k_x^{(1,2)} = \frac{k_y \varepsilon_{xy} \pm \sqrt{(\varepsilon_{xy}^2 - \varepsilon_{xx} \varepsilon_{yy})(k_y^2 - k_0^2 \varepsilon_{xx})}}{\varepsilon_{xx}} \]
Transverse momentum conservation
Transverse momentum conservation

Below $\theta_t$, propagative mode excited
Transverse momentum conservation

Below $\theta_t$, propagative mode excited
Transverse momentum conservation

Below $\theta_t$, propagative mode excited

Above $\theta_t$, evanescent mode excited
Below $\theta_t$, propagative mode excited

Above $\theta_t$, evanescent mode excited
Reflection map

Evanescent mode

Fabry-Pérot
Exact solution (without losses in metal)

\[
\cos (k_y D) = \frac{(\kappa_d \varepsilon_m + \kappa_m \varepsilon_d)^2}{4 \kappa_d \kappa_m \varepsilon_d \varepsilon_m} \cosh (\kappa_d d_d + \kappa_m d_m) - \frac{(\kappa_d \varepsilon_m - \kappa_m \varepsilon_d)^2}{4 \kappa_d \kappa_m \varepsilon_d \varepsilon_m} \cosh (\kappa_d d_d - \kappa_m d_m)
\]
Exact solution (without losses in metal)

\[ \cos (k_y D) = \frac{\left(\kappa_d \varepsilon_m + \kappa_m \varepsilon_d\right)^2}{4 \kappa_d \kappa_m \varepsilon_d \varepsilon_m} \cosh (\kappa_d d_d + \kappa_m d_m) - \frac{\left(\kappa_d \varepsilon_m - \kappa_m \varepsilon_d\right)^2}{4 \kappa_d \kappa_m \varepsilon_d \varepsilon_m} \cosh (\kappa_d d_d - \kappa_m d_m) \]

- Periodic isofrequency curve for the propagative mode
- Close isofrequency curve for the evanescent mode
Transverse momentum conservation ($k_y = 0$)

Always a propagative and evanescent mode excited!
Transverse momentum conservation

Always a propagative and evanescent mode excited!
Always a propagative and evanescent mode excited!

→ Interference at the output
Fano resonances ($\Theta = 45^\circ$)

Evanescent background

Reflectance

$B$ (nm)
Fano resonances ($\Theta = 45^\circ$)

Reflectance

$B$ (nm)

Evanescent background

Fabry-Pérot
Fano resonances ($\Theta = 45^\circ$)

- Evanescent background
- Fabry-Pérot
- Exact reflection
Spectrum for $B = 5$ nm
Spectrum for $B = 35$ nm
Reflection map (without loss)

\[ 2k_x(\theta)B + 2\varphi(\theta) = 2\pi m \] Phase matching

Lossy metal: condition for Fano resonances

\[ \frac{\text{Re}(n_x^{\text{prop}})}{\text{Im}(n_x^{\text{prop}})} \]

\[ \text{Im}(n_x^{\text{evan}}) \]
Lossy metal: condition for Fano resonances

- Propagating mode should have large real part and small imaginary part of refractive effective index
- Evanescent mode should have imaginary part not too high (background would disappear) and not too low (background not efficient)
Lossy metal: conditions for Fano resonances

- Propagating mode should have large real part and small imaginary part of refractive effective index
- Evanescent mode should have imaginary part not too high (background would disappear) and not too low (background not efficient)
Scattering with losses for $\Theta = 65^\circ$

Fano resonances still present but more or less damped
Introduction of gain in the dielectric: \( \text{Im}(n_{TiO_2}) = -0.07 \)
Comparison lossless – gain/loss structures

Introduction of gain allows 100% transmittance-reflectance Fano resonances.
Actually difficult to introduce gain in TiO2.
Would be easier to work with semiconductors in infrared regime.
Conclusions

- Hyperbolic metamaterials are periodic plasmonic structures with positive component of dielectric tensor in one direction and negative in another

- Fano resonances in ultra compact cavities for great control of the reflection and transmission of light

- Effective medium approximation inaccurate for this work. Predicts the excitation of one single mode, no Fano resonances possible

- Other topics: Heat transfer, active HMM, tunable HMM with graphene, homogenization theory, ...
Thank you for your attention

This work is financially supported by the F.R.I.A.-F.N.R.S.