Surface Impedance Boundary Condition with Circuit Coupling for the 3D Finite Element Modeling of Wireless Power Transfer

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Abstract—A light 3D finite element magnetodynamic a-v model of resonant wireless power transfer coils using 3D surface impedance boundary condition strongly coupled with an external circuit is proposed, reflecting the importance of external circuit elements (notably capacitances) in the resonance phenomena. Preliminary results outline the accuracy of the model.

Index Terms—Finite Element, Inductive Power Transfer

I. CONTEXT AND OBJECTIVES

The 3D finite element model of the coils used for resonant Wireless Power Transfer (WPT) as massive conductors is submitted to important computational burden. The skin and proximity effects appearing at the working frequency level (tens of kHz for the transfer of high power) require a high refinement of the conductors’ volume mesh, making massive conductors formulations hardly applicable. Here, profit is taken from a 3D Surface Impedance Boundary Condition with Circuit Coupling for Inductive Power Transfer (—Finite Element, Inductive Power Transfer—) [1] to avoid the volume mesh inside the conductors and relaxing the computational constraints. Reflecting the important influence of the external circuit on the power transfer (through the resonant effect), a natural circuit coupling is applied leading to a new way to implement SIBC.

II. METHODOLOGY

The methodology is based on the extension of [2] to SIBCs. The magnetodynamic a-v weak formulation (with a the magnetic vector potential and v the electric scalar potential) is obtained from the weak form of the Ampère’s equation:

\[
\left( \mu^{-1} \nabla \times a \cdot \nabla \times a' \right)_{\Omega} + (n \times h, a')_{\Gamma} = \sigma (\partial_t a + \nabla v, a')_{\Omega_c} = 0 \quad \forall \ a' \in H^1(\Omega) \tag{1}
\]

where \( \Omega \) is the whole domain (of boundary \( \Gamma \)), \( \Omega_c \) is the massive conductors domain, \( H^1(\Omega) \) is a curl-conform function space defined on \( \Omega \), \( n \) is the unit normal vector exterior to \( \Omega \), \( h \) is the magnetic field, \( \sigma \) is the conductivity and \( \mu \) is the permeability. \( (.,.)_{\Omega} \) and \( (.,.)_{\Gamma} \) denote respectively a volume integral in \( \Omega \) and a surface integral on \( \Gamma \) of the product of their arguments. Massive conductors \( \Omega_c \) can be extracted from \( \Omega \) by imposing an SIBC on their boundary (denoted by \( \Gamma_c \)):

\[
\left[n \times h \right]_{\Gamma_c} = Z_c^{-1} (n \times (\partial_t a + \nabla v)) \times n \mid_{\Gamma_c} \tag{2}
\]

with \( Z_c \) the first order surface impedance of conductors \( \Omega_c \):

\[
Z_c = (\sigma \delta)^{-1} (1 + j) \quad \text{with} \quad \delta = \sqrt{2/(\sigma \mu)} \tag{3}
\]

where \( \omega = 2\pi f \) is the angular frequency and \( j \) is the imaginary unit. The coupling of a conductor \( i \) with an external circuit is implemented by defining two infinitely close electrodes modeled as a single contour cut \( \gamma_i \) in the tube modeling the conductor. Let \( V_i \) and \( I_i \) be respectively the voltage and the current between and across the electrodes. \( V_i \) is strongly determined by defining \( v \) as \( v = \sum_{i \in \gamma_i} V_i v_{i,s} \) with \( v_{i,s} \) a basis function supported here by the surface mesh of \( \Gamma_i \) only, equal to 1 on one side of \( \gamma_i \) and decreasing linearly to 0 in the first adjacent layer of elements of \( \Gamma_i \). The introduction of the SIBC and of this particular definition of \( v \) in (1) leads to the final weak form, whose unknowns are the potential \( a \) and the global voltage \( V_i \) of each conductor, \( \forall \ a' \in H^1(\Omega) \):

\[
\left( \mu^{-1} \nabla \times a \cdot \nabla \times a' \right)_{\Omega} + (n \times h, a')_{\Gamma} \\
+ \left[Z_c^{-1} n \times \nabla t, a \cdot a' \right]_{\Gamma_c} \\
+ \sum_{i \in \gamma_i} V_i \left[Z_c^{-1} n \times \nabla v_i, a \cdot a' \right]_{\Gamma_i} = 0. \tag{4}
\]

The current \( I_i \) is weakly obtained by taking \( a' = \nabla v_{i,s} \):

\[
I_i = \left[Z_c^{-1} n \times \nabla t, a \cdot a' \right]_{\Gamma_i} \\
+ \sum_{i \in \gamma_i} V_i \left[Z_c^{-1} n \times \nabla v_i, a \cdot a' \right]_{\Gamma_i}. \tag{5}
\]

III. PRELIMINARY RESULTS

Fig. 1 shows the good accuracy of the method for determining a mutual inductance using circuit relations despite using drastically less mesh elements. In the full paper, the application to a full WPT device in resonant state is targeted.

Fig. 1. Mutual inductance extracted from circuit relations for different distances between two identical and aligned 2-turns coils at \( f = 100 \) kHz

REFERENCES


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