Journey planning in uncertain environments, the multi-objective way

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Think tank “Systèmes complexes”
Aim of this talk

Flavor of ≠ types of **useful strategies** in stochastic environments.

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Applications to the shortest path problem.

Find a path of minimal length in a weighted graph (Dijkstra, Bellman-Ford, etc) [CGR96].
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Flavor of ≠ types of **useful strategies** in stochastic environments.


Applications to the **shortest path problem**.

What if the environment is **uncertain**? E.g., in case of heavy traffic, some roads may be crowded.
Planning a journey in an uncertain environment

Each action takes time, target = work.

▷ What kind of strategies are we looking for when the environment is stochastic (Markov decision process)?
Solution 1: minimize the *expected* time to work

- “Average” performance: meaningful when you journey often.
- **Simple strategies** suffice: no memory, no randomness.
- Taking the **car** is optimal: $\mathbb{E}^\sigma_D(\text{TS}^{\text{work}}) = 33$. 
Solution 2: traveling without taking too many risks

Minimizing the *expected time* to destination makes sense if we travel often and it is not a problem to be late.

With car, in 10% of the cases, the journey takes 71 minutes.
Solution 2: traveling without taking too many risks

Most bosses will not be happy if we are late too often... what if we are risk-averse and want to avoid that?
Solution 2: maximize the *probability* to be on time

**Specification:** reach work within 40 minutes with 0.95 probability
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**Sample strategy:** take the train $\sim P_D^{\sigma}[T_{S_{work}} \leq 40] = 0.99$

**Bad choices:** car (0.9) and bike (0.0)
Solution 3: strict worst-case guarantees

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**Sample strategy:** bike \(\sim\) worst-case reaching time = 45 minutes.

**Bad choices:** train \((wc = \infty)\) and car \((wc = 71)\)
Solution 3: strict worst-case guarantees

Worst-case analysis \(\leadsto\) two-player game against an antagonistic adversary (bad guy)

▷ forget about probabilities and give the choice of transitions to the adversary
Solution 4: minimize the *expected* time under strict worst-case guarantees

- **Expected time:** car $\sim E = 33$ but $wc = 71 > 60$
- **Worst-case:** bike $\sim wc = 45 < 60$ but $E = 45 >>> 33$
Solution 4: minimize the *expected* time under strict worst-case guarantees

In practice, we want both! Can we do better?

- **Beyond worst-case synthesis** [BFRR17]: minimize the expected time under the worst-case constraint.
Solution 4: minimize the *expected* time under strict worst-case guarantees

Sample strategy: try train up to 3 delays then switch to bike.

\[ wc = 58 < 60 \quad \text{and} \quad E \approx 37.34 \ll 45 \]

\[ \sim \quad \text{Strategies need memory} \quad \text{\sim more complex!} \]
Solution 5: multiple objectives $\Rightarrow$ trade-offs

Two-dimensional weights on actions: time and cost.

Often necessary to consider trade-offs: e.g., between the probability to reach work in due time and the risks of an expensive journey.
Solution 5: multiple objectives ⇒ trade-offs

Solution 2 (probability) can only ensure a **single constraint**.

- **C1**: 80% of runs reach work in at most 40 minutes.
  - Taxi $\sim$ $\leq$ 10 minutes with probability $0.99 > 0.8$. 
Solution 5: multiple objectives ⇒ trade-offs

Solution 2 (probability) can only ensure a single constraint.

- **C1**: 80% of runs reach work in at most 40 minutes.
  - Taxi \(\sim\) ≤ 10 minutes with probability \(0.99 \geq 0.8\).
- **C2**: 50% of them cost at most 10$ to reach work.
  - Bus \(\sim\) ≥ 70% of the runs reach work for 3$. 
Solution 5: multiple objectives ⇒ trade-offs

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Taxi $\not\models C2$, bus $\not\models C1$. What if we want $C1 \land C2$?
Solution 5: multiple objectives $\Rightarrow$ trade-offs

- **C1**: 80% of runs reach work in at most 40 minutes.
- **C2**: 50% of them cost at most 10 to reach work.

Study of multi-constraint percentile queries [RRS17].

- Sample strategy: bus once, then taxi. Requires *memory*.
- Another strategy: bus with probability 3/5, taxi with probability 2/5. Requires *randomness*.
Solution 5: multiple objectives ⇒ trade-offs

- C1: 80% of runs reach work in at most 40 minutes.
- C2: 50% of them cost at most 10$ to reach work.

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In general, both memory and randomness are required.

≠ previous problems → more complex!
Conclusion

Our research aims at:
- defining meaningful strategy concepts and objectives,
- providing algorithms and tools to compute those strategies,
- classifying the complexity of the different problems from a theoretical standpoint.

→ Is it mathematically possible to obtain efficient algorithms?
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- defining meaningful *strategy concepts* and *objectives*,
- providing *algorithms* and *tools* to compute those strategies,
- classifying the *complexity* of the different problems from a theoretical standpoint.

→ Is it mathematically possible to obtain efficient algorithms?

Thank you! Any question?
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