Average-Energy Games

Patricia Bouyer-Decitre\textsuperscript{1}  Kim G. Larsen\textsuperscript{2}  Simon Laursen\textsuperscript{2}  Nicolas Markey\textsuperscript{1}  Mickael Randour\textsuperscript{1}

\textsuperscript{1}LSV - CNRS & ENS Cachan  \textsuperscript{2}Aalborg University

12.03.2015 - ERC Workshop - IST Austria
General context: strategy synthesis in quantitative games

1. How complex is it to decide if a winning strategy exists?
2. How complex such a strategy needs to be? Simpler is better.
3. Can we synthesize one efficiently?

⇒ Depends on the winning objective.
The talk in one slide

- **New quantitative objective**
  - Total-payoff (TP) “refines” mean-payoff (MP) (MP value = 0)
  - Average-energy (AE) “refines” TP
The talk in one slide

- **New quantitative objective**
  - Total-payoff (TP) “refines” mean-payoff (MP) (MP value = 0)
  - Average-energy (AE) “refines” TP
  - characterizes the **average energy level** along an infinite play

![Energy Graphs](image-url)
The talk in one slide

- **New quantitative objective**
  - Total-payoff (TP) “refines” mean-payoff (MP) (MP value = 0)
  - **Average-energy (AE) “refines” TP**
  - characterizes the **average energy level** along an infinite play

- Conjunction with **energy constraints**: lower and/or upper bounds on the energy level (e.g., fuel tank)
The talk in one slide

- **New quantitative objective**
  - ▶ Total-payoff (TP) “refines” mean-payoff (MP) (MP value = 0)
  - ▶ **Average-energy (AE)** “refines” TP
  - ↦ characterizes the **average energy level** along an infinite play

- Conjunction with **energy constraints**: lower and/or upper bounds on the energy level (e.g., fuel tank)

**Ongoing work!**
<table>
<thead>
<tr>
<th></th>
<th>Context &amp; Definitions</th>
<th>Average-Energy Games</th>
<th>Average-Energy with Energy Constraints</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Context &amp; Definitions</td>
<td>Average-Energy Games</td>
<td>Average-Energy with Energy Constraints</td>
<td>Conclusion</td>
</tr>
<tr>
<td>2</td>
<td>Average-Energy Games</td>
<td>Average-Energy Games</td>
<td>Average-Energy with Energy Constraints</td>
<td>Conclusion</td>
</tr>
<tr>
<td>3</td>
<td>Average-Energy with Energy Constraints</td>
<td>Average-Energy Games</td>
<td>Average-Energy with Energy Constraints</td>
<td>Conclusion</td>
</tr>
<tr>
<td>4</td>
<td>Conclusion</td>
<td>Average-Energy Games</td>
<td>Average-Energy with Energy Constraints</td>
<td>Conclusion</td>
</tr>
</tbody>
</table>
1. Context & Definitions

2. Average-Energy Games

3. Average-Energy with Energy Constraints

4. Conclusion
Two-player turn-based games on graphs

- $G = (S_1, S_2, T, w)$
- $S = S_1 \cup S_2, S_1 \cap S_2 = \emptyset, T \subseteq S \times S, \ w : T \rightarrow \mathbb{Z}$
- $\mathcal{P}_1$ states =
- $\mathcal{P}_2$ states =
- Plays have values
  - $f : \text{Plays}(G) \rightarrow \mathbb{R} \cup \{-\infty, \infty\}$
- Players follow **pure** strategies
  - $\sigma_i : \text{Prefs}_i(G) \rightarrow S$
Energy, total-payoff, mean-payoff

\[ EL(\pi(n)) = \sum_{i=0}^{n-1} w(s_i, s_{i+1}) \]

\[ TP(\pi) = \limsup_{n \to \infty} EL(\pi(n)) \]

\[ MP(\pi) = \limsup_{n \to \infty} \frac{1}{n} EL(\pi(n)) \]
Energy, total-payoff, mean-payoff

- $EL(\pi(n)) = \sum_{i=0}^{n-1} w(s_i, s_{i+1})$
- $TP(\pi) = \limsup_{n \to \infty} EL(\pi(n))$
- $MP(\pi) = \limsup_{n \to \infty} \frac{1}{n} EL(\pi(n))$
Energy, total-payoff, mean-payoff

Energy, total-payoff, mean-payoff

\[ EL(\pi(n)) = \sum_{i=0}^{n-1} w(s_i, s_{i+1}) \]

\[ \overline{TP}(\pi) = \limsup_{n \to \infty} EL(\pi(n)) \]

\[ \overline{MP}(\pi) = \limsup_{n \to \infty} \frac{1}{n} EL(\pi(n)) \]
Energy, total-payoff, mean-payoff

- $EL(\pi(n)) = \sum_{i=0}^{n-1} w(s_i, s_{i+1})$
- $\overline{TP}(\pi) = \limsup_{n \to \infty} EL(\pi(n))$
- $\overline{MP}(\pi) = \limsup_{n \to \infty} \frac{1}{n} EL(\pi(n))$
Energy, total-payoff, mean-payoff

\[ EL(\pi(n)) = \sum_{i=0}^{n-1} w(s_i, s_{i+1}) \]

\[ TP(\pi) = \limsup_{n \to \infty} EL(\pi(n)) \]

\[ MP(\pi) = \limsup_{n \to \infty} \frac{1}{n} EL(\pi(n)) \]
Energy, total-payoff, mean-payoff

\[ EL(\pi(n)) = \sum_{i=0}^{n-1} w(s_i, s_{i+1}) \]

\[ TP(\pi) = \limsup_{n \to \infty} EL(\pi(n)) \]

\[ MP(\pi) = \limsup_{n \to \infty} \frac{1}{n} EL(\pi(n)) \]
Energy, total-payoff, mean-payoff

\[ EL(\pi(n)) = \sum_{i=0}^{n-1} w(s_i, s_{i+1}) \]

\[ TP(\pi) = \limsup_{n \to \infty} EL(\pi(n)) \]

\[ MP(\pi) = \limsup_{n \to \infty} \frac{1}{n} EL(\pi(n)) \]

Then, \((2, 5, 2)^\omega\)
Decision problems

- **TP (MP) threshold problem**
  
  $\triangleright$ Given $t \in \mathbb{Q}$ and $s_{\text{init}} \in S$, $\exists\sigma_1 \in \Sigma_1$ s.t. $\forall\sigma_2 \in \Sigma_2$, $TP(\text{Outcome}(s_{\text{init}}, \sigma_1, \sigma_2)) \leq t$

  $\hookrightarrow$ we take the minimizer point of view

- **Lower-bounded energy problem**

  $\triangleright$ Given $c_{\text{init}} \in \mathbb{N}$ and $s_{\text{init}} \in S$, $\exists\sigma_1 \in \Sigma_1$ s.t. $\forall\sigma_2 \in \Sigma_2$, $\forall n \geq 0$, $c_{\text{init}} + EL(\text{Outcome}(s_{\text{init}}, \sigma_1, \sigma_2)(n)) \geq 0$

  $\hookrightarrow$ fixed initial credit

- **Lower- and upper-bounded energy problem**

  $\triangleright$ Given $c_{\text{init}} \in \mathbb{N}$, $U \in \mathbb{N}$ and $s_{\text{init}} \in S$, $\exists\sigma_1 \in \Sigma_1$ s.t. $\forall\sigma_2 \in \Sigma_2$, $\forall n \geq 0$, $c_{\text{init}} + EL(\text{Outcome}(s_{\text{init}}, \sigma_1, \sigma_2)(n)) \in [0, U]$
### Known results

<table>
<thead>
<tr>
<th>Game objective</th>
<th>1-player</th>
<th>2-player</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MP$</td>
<td>P [Kar78]</td>
<td>NP ∩ coNP [ZP96]</td>
</tr>
<tr>
<td>$TP$</td>
<td>P [FV97]</td>
<td>NP ∩ coNP [GS09]</td>
</tr>
<tr>
<td>$EG_L$</td>
<td>P [BFL+08]</td>
<td>NP ∩ coNP [CdAHS03, BFL+08]</td>
</tr>
<tr>
<td>$EG_{LU}$</td>
<td>PSPACE-c. [FJ13]</td>
<td>EXPTIME-c. [BFL+08]</td>
</tr>
</tbody>
</table>

For all objectives but $EG_{LU}$, **memoryless** strategies suffice for both players.
Average-energy: motivating example

**HYDAC** oil pump industrial case study [CJL⁺09] (Quasimodo research project).

**Goals:**

1. Keep the oil level in the safe zone. 
   \[ \leftrightarrow \ EG_{LU} \]

2. Minimize the average oil level. 
   \[ \leftrightarrow \ AE \]

\[ \Rightarrow \] Conjunction: \( AE_{LU} \)
1. Context & Definitions

2. Average-Energy Games

3. Average-Energy with Energy Constraints

4. Conclusion
Average-energy: definition

Recall

- $EL(\pi(n)) = \sum_{i=0}^{n-1} w(s_i, s_{i+1})$
- $TP(\pi) = \limsup_{n \to \infty} EL(\pi(n))$
- $MP(\pi) = \limsup_{n \to \infty} \frac{1}{n} EL(\pi(n))$

+ infimum variants $TP$, $MP$, $AE$

Average-energy (AE)

Describes the average energy level along a play:

$$\overline{AE}(\pi) = \limsup_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n} EL(\pi(i))$$
TP “refines” MP

- If $\mathcal{P}_1$ (minimizer) can ensure $\overline{MP} = \overline{MP} < 0$ (memoryless), he can ensure $\overline{TP} = \overline{TP} = -\infty$.
- If $\mathcal{P}_2$ (maximizer) can ensure $\overline{MP} = \overline{MP} > 0$ (memoryless), he can ensure $\overline{TP} = \overline{TP} = \infty$. 
TP “refines” MP

- If $P_1$ (minimizer) can ensure $MP = MP < 0$ (memoryless), he can ensure $TP = TP = -\infty$.
- If $P_2$ (maximizer) can ensure $MP = MP > 0$ (memoryless), he can ensure $TP = TP = \infty$.

$\Rightarrow$ **TP discriminates “MP-zero” strategies** depending on the high points ($TP$) or low points ($TP$) of cycles.

$\overline{MP} = \overline{MP} = 0$

$\overline{TP} = \overline{TP} = 1$

$\overline{MP} = \overline{MP} = 0$

$\overline{TP} = 3$, $TP = -1$
AE “refines” TP

AE describes the long-run average EL

By definition, \( AE(\pi), \overline{AE}(\pi) \in [TP(\pi), \overline{TP}(\pi)] \).
AE “refines” TP

AE describes the **long-run average EL**

\[ AE(\pi), \overline{AE}(\pi) \in [TP(\pi), \overline{TP}(\pi)] \]

⇒ AE discriminates strategies with identical high/low points.

Identical MP and TP, but AE lower in the first one.
Memoryless determinacy (1/2)

Classical criteria from the literature cannot be applied out-of-the-box [EM79, BSV04, AR14, GZ04, Kop06].

- Common approach: connect first cycle games and infinite-duration ones.

- Requires e.g., closure under cyclic permutation and concatenation [AR14].

Intuitively: ability to mix and shuffle good cycles and stay good.
Memoryless determinacy (1/2)

Classical criteria from the literature cannot be applied out-of-the-box [EM79, BSV04, AR14, GZ04, Kop06].

→ Common approach: connect first cycle games and infinite-duration ones.

→ Requires e.g., closure under cyclic permutation and concatenation [AR14].

Intuitively: ability to mix and shuffle good cycles and stay good.

Not true in general for AE!

\[ C_1 = \{-1\}, \quad C_2 = \{1\}, \quad C_3 = \{1, -2\} \]

\[ AE(C_1C_2) = (−1 + 0)/2 = −1/2 < AE(C_2C_1) = (1 − 0)/2 = 1/2 \]

\[ AE(C_3) = 0 \text{ but } AE(C_3C_3) = −1/2 < 0 \]
Memoryless determinacy (1/2)

Classical criteria from the literature cannot be applied out-of-the-box [EM79, BSV04, AR14, GZ04, Kop06].

Common approach: connect first cycle games and infinite-duration ones.

Requires e.g., closure under cyclic permutation and concatenation [AR14].

Intuitively: ability to mix and shuffle good cycles and stay good.

We can only shuffle/repeat cycles that are neutral w.r.t. the energy level!

→ zero-cycles
Memoryless determinacy (2/2)

Two key properties:

1. **Extraction of prefixes**
   - Let $\rho \in \text{Prefs}(G)$, $\pi \in \text{Plays}(G)$. Then,
     $$\overline{\text{AE}}(\rho \cdot \pi) = \text{EL}(\rho) + \overline{\text{AE}}(\pi).$$

2. **Extraction of a best cycle**
   - Given an infinite sequence of zero-cycles, one can select and repeat a *best cycle* to minimize the average-energy.
One-player games: strategy

Sketch (minimizer)

1. If you can ensure $MP < 0$, do it.
   - Memoryless [EM79], implies $AE = -\infty$.

2. If you cannot ensure $MP = 0$, forget it.
   - You are doomed, $AE = \infty$.

3. Play the strategy that minimizes

$$\overline{AE}(\rho \cdot C^\omega) = EL(\rho) + \overline{AE}(C),$$

where $C$ is a simple zero-cycle.

Picking the best combination can be done without memory.
One-player games: P algorithm (1/2)

- Case $MP < 0$ is easy
  - Look for a negative cycle (e.g., Bellman-Ford, $O(|S|^3)$)
One-player games: P algorithm (1/2)

- Case $MP < 0$ is easy
  - Look for a negative cycle (e.g., Bellman-Ford, $O(|S|^3)$)

- Assume $MP = 0$: pick the best combination of $\rho$ and $C$
  - Computing the best $\rho$ for each state is easy with classical graph algorithms (e.g., Bellman-Ford).
  - Main task: computing the best $C$ (AE-wise) for each state.
One-player games: P algorithm (1/2)

- **Case** $MP < 0$ is easy
  - Look for a negative cycle (e.g., Bellman-Ford, $O(|S|^3)$)

- **Assume** $MP = 0$: **pick the best combination** of $\rho$ and $C$
  - Computing the best $\rho$ for each state is easy with classical graph algorithms (e.g., Bellman-Ford).
  - **Main task**: computing the best $C$ (AE-wise) for each state.

- For each state, we compute the best cycle of length $k$, for all $k \in \{1, \ldots, |S|\}$, then pick the best one.
  - Need to compute $C_{s,k}$ in polynomial time.
One-player games: P algorithm (2/2)

Computing $C_{s,k}$: build a new graph $G_{s,k}$ of size $|S| \cdot (k + 1)$. 

$$
(s, k) \rightarrow (s', k - 1) \rightarrow (s'', k - 1) \rightarrow (s''', k - 2) \rightarrow (s', 0)
$$

$$
(s, k) \rightarrow (s', k - 1) \rightarrow (s'', k - 1) \rightarrow (s''', k - 2) \rightarrow (s', 0)
$$
One-player games: P algorithm (2/2)

Computing $C_{s,k}$: build a **new graph** $G_{s,k}$ of size $|S| \cdot (k + 1)$.

Two weights by edge: $(c, l \cdot c)$

$c$ cost, $l$ origin level
One-player games: P algorithm (2/2)

Computing $C_{s,k}$: build a new graph $G_{s,k}$ of size $|S| \cdot (k + 1)$.

Two weights by edge: $(c, l \cdot c)$
$c$ cost, $l$ origin level

Write an LP s.t.
solution is a path from $(s, k)$ to $(s, 0)$

$\sum_{1st \ dim.} = 0$ (zero cycle)

minimize $\sum_{2nd \ dim.} = AE(C) \cdot k$
One-player games: P algorithm (2/2)

Computing $C_{s,k}$: build a **new graph** $G_{s,k}$ of size $|S| \cdot (k + 1)$.

⇒ polynomial-time algorithm for 1-p. games

Write an LP s.t.

solution is a path from $(s, k)$ to $(s, 0)$

$$
\sum_{1st \ dim.} = 0 \; (\text{zero cycle})
$$

minimize $$
\sum_{2nd \ dim.} = AE(C) \cdot k
$$
Two-player games

- **Memoryless determinacy**
  - Follows from the 1-p. results (minimizer *and* maximizer) using Gimbert and Zielonka [GZ05].

- **Threshold problem in** $\text{NP} \cap \text{coNP}$.  
  - Memoryless determinacy + P for one-player games.

- **“Mean-payoff” hard.**  
  - Replace any edge of weight $c$ by two consecutive edges of values $2 \cdot c$ and $-2 \cdot c$.  
  - $MP(\pi)$ in $G = AE(\pi)$ in $G'$. 

### Wrap-up

<table>
<thead>
<tr>
<th>Game objective</th>
<th>1-player</th>
<th>2-player</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MP$</td>
<td>$P$ [Kar78]</td>
<td>$NP \cap \text{coNP}$ [ZP96]</td>
</tr>
<tr>
<td>$TP$</td>
<td>$P$ [FV97]</td>
<td>$NP \cap \text{coNP}$ [GS09]</td>
</tr>
<tr>
<td>$EG_L$</td>
<td>$P$ [BFL$^+$08]</td>
<td>$NP \cap \text{coNP}$ [CdAHS03, BFL$^+$08]</td>
</tr>
<tr>
<td>$EG_{LU}$</td>
<td>PSPACE-c. [FJ13]</td>
<td>EXPTIME-c. [BFL$^+$08]</td>
</tr>
<tr>
<td>$AE$</td>
<td>$P$</td>
<td>$NP \cap \text{coNP}$</td>
</tr>
</tbody>
</table>

▷ For all objectives but $EG_{LU}$, *memoryless* strategies suffice for both players.
1. Context & Definitions

2. Average-Energy Games

3. Average-Energy with Energy Constraints

4. Conclusion
Two settings

1. $AE_{LU}$: AE with lower (0) and upper ($U \in \mathbb{N}$) bounds.

2. $AE_L$: AE with only the lower bound (0).

→ Fixed initial credit $c_{\text{init}} = 0$. 
Memory is needed!

Example: $AE_L \sim$ minimize $AE$ while keeping $EL \geq 0$. 

(a) 1-Player $AE_L$ game

(b) Taking the +1 loop

(c) Taking +4 then +1

(d) Taking the +4 loop
Memory is needed!

Example: $AE_L \sim$ minimize $AE$ while keeping $EL \geq 0$.

Non trivial behavior in general!
→ Need to choose carefully which cycles to play.
\( AE_{LU} \) problem: reduction to \( AE \)

- Expanded graph constraining the game within the energy bounds \([0, U]\). **Pseudo-polynomial size:** \( \mathcal{O}(|S| \cdot (U + 1)) \).
- If we go out, \( AE = \infty \).

\[ P_1 \] minimizes \( AE \) and maintains \( EL \in [0, 2] \) in the left game iff \( P_1 \) minimizes \( AE \) in the right game.
### $AE_{LU}$ problem: complexity

<table>
<thead>
<tr>
<th>Game objective</th>
<th>1-player</th>
<th>2-player</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MP$</td>
<td>P [Kar78]</td>
<td>NP ∩ coNP [ZP96]</td>
</tr>
<tr>
<td>$TP$</td>
<td>P [FV97]</td>
<td>NP ∩ coNP [GS09]</td>
</tr>
<tr>
<td>$EG_L$</td>
<td>P [BFL+08]</td>
<td>NP ∩ coNP [CdAHS03, BFL+08]</td>
</tr>
<tr>
<td>$EG_{LU}$</td>
<td>PSPACE-c. [FJ13]</td>
<td>EXPTIME-c. [BFL+08]</td>
</tr>
</tbody>
</table>

| AE | P | NP ∩ coNP |
| $AE_{LU}$ (poly. $U$) | P | NP ∩ coNP |
| $AE_{LU}$ (arbitrary) | EXPTIME /PSPACE-h. | NEXPTIME ∩ coNEXPTIME /EXPTIME-h. |

- Lower bounds follow from $EG_{LU}$.
- Memory is required (at most exponential).
**AE_{L}** problem: one-player case

**Key argument:** (upper) bounding the value of the energy over a witness winning path.

- It is not necessary to accumulate *too much* energy. Intuitively, otherwise we cannot keep the AE sufficiently low.

- Bound $U$ polynomial in $|S|$, the largest absolute weight $W$ and the threshold $t$ for the AE constraint.

- **Reduction to an AE_{LU} problem.**

- EXPTIME-algorithm.
$AE_L$ problem: one-player case

**Key argument:** (upper) bounding the value of the energy over a witness winning path.

- It is not necessary to accumulate *too much* energy. Intuitively, otherwise we cannot keep the $AE$ sufficiently low.
- Bound $U$ polynomial in $|S|$, the largest absolute weight $W$ and the threshold $t$ for the $AE$ constraint.
- **Reduction to an $AE_{LU}$ problem.**
- EXPTIME-algorithm.

**Lower bound:** NP-hard via *subset sum problem* [GJ79].

- Find a subset of a set of naturals s.t. the sum of its elements is exactly equal to target $T \in \mathbb{N}$.
- The energy LB can be used to ensure a sum $\geq T$ and the AE to ensure $\leq T$. 
$AE_L$ problem: two-player case

- Algorithm: work still in progress.
  - We believe we can apply a similar approach, upper bounding the energy.
  - Non-trivial alternations between carefully chosen cycles is required (see previous example).
**$AE_L$ problem: two-player case**

- **Algorithm:** work still in progress.
  - We believe we can apply a similar approach, upper bounding the energy.
  - Non-trivial alternations between carefully chosen cycles is required (see previous example).

- Problem is **EXPTIME-hard via countdown games** [JSL08].
### $AE_L$ problem: complexity

<table>
<thead>
<tr>
<th>Game objective</th>
<th>1-player</th>
<th>2-player</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MP$</td>
<td>P [Kar78]</td>
<td>NP ∩ coNP [ZP96]</td>
</tr>
<tr>
<td>$TP$</td>
<td>P [FV97]</td>
<td>NP ∩ coNP [GS09]</td>
</tr>
<tr>
<td>$EG_L$</td>
<td>P [BFL+08]</td>
<td>NP ∩ coNP [CdAHS03, BFL+08]</td>
</tr>
<tr>
<td>$EG_{LU}$</td>
<td>PSPACE-c. [FJ13]</td>
<td>EXPTIME-c. [BFL+08]</td>
</tr>
<tr>
<td>$AE$</td>
<td>P</td>
<td>NP ∩ coNP</td>
</tr>
<tr>
<td>$AE_{LU}$ (poly. $U$)</td>
<td>P</td>
<td>NP ∩ coNP</td>
</tr>
<tr>
<td>$AE_{LU}$ (arbitrary)</td>
<td>EXPTIME /PSPACE-h.</td>
<td>NEXPTIME ∩ coNEXPTIME /EXPTIME-h.</td>
</tr>
<tr>
<td>$AE_L$</td>
<td>EXPTIME/NP-h.</td>
<td>??? /EXPTIME-h.</td>
</tr>
</tbody>
</table>

▶ Memory is required (at most exponential).
1. Context & Definitions

2. Average-Energy Games

3. Average-Energy with Energy Constraints

4. Conclusion
Wrap-up

New quantitative objective.

- Practical motivations (e.g., HYDAC).
- “Refines” TP (and MP).
- Same complexity class as $E_{GL}$, $MP$ and $TP$ games.
- Still some open questions.
  - Complexity gaps.
  - Algorithm for 2-player $AE_L$ games.
Thanks!

Do not hesitate to discuss with us!
References I

Benjamin Aminof and Sasha Rubin.
First cycle games.

Patricia Bouyer, Uli Fahrenberg, Kim Gulstrand Larsen, Nicolas Markey, and Jiří Srba.
Infinite runs in weighted timed automata with energy constraints.

Henrik Björklund, Sven Sandberg, and Sergei Vorobyov.
Memoryless determinacy of parity and mean payoff games: A simple proof.

Arindam Chakrabarti, Luca de Alfaro, Thomas A. Henzinger, and Mariëlle Stoelinga.
Resource interfaces.

Krishnendu Chatterjee, Laurent Doyen, Mickael Randour, and Jean-François Raskin.
Looking at mean-payoff and total-payoff through windows.
Franck Cassez, Jan J. Jensen, Kim Gulstrand Larsen, Jean-François Raskin, and Pierre-Alain Reynier.  
Automatic synthesis of robust and optimal controllers – an industrial case study.  

Andrzej Ehrenfeucht and Jan Mycielski.  
Positional strategies for mean payoff games.  

John Fearnley and Marcin Jurdziński.  
Reachability in two-clock timed automata is PSPACE-complete.  
In Fedor V. Fomin, Rusins Freivalds, Marta Kwiatkowska, and David Peleg, editors, Proceedings of the 40th International Colloquium on Automata, Languages and Programming (ICALP’13) – Part II, volume 7966 of Lecture Notes in Computer Science, pages 212–223. Springer-Verlag, July 2013.

Jerzy Filar and Koos Vrieze.  
Competitive Markov decision processes.  

Michael R. Garey and David S. Johnson.  
References III

Thomas Gawlitza and Helmut Seidl.
Games through nested fixpoints.

Hugo Gimbert and Wiesław Zielonka.
When can you play positionnaly?

Hugo Gimbert and Wiesław Zielonka.
Games where you can play optimally without any memory.
In Martín Abadi and Luca de Alfaro, editors, Proceedings of the 16th International Conference on Concurrency Theory (CONCUR’05), volume 3653 of Lecture Notes in Computer Science, pages 428–442. Springer-Verlag, August 2005.

Marcin Jurdziński, Jeremy Sproston, and François Laroussinie.
Model checking probabilistic timed automata with one or two clocks.

Richard M. Karp.
A characterization of the minimum cycle mean in a digraph.
Eryk Kopczynski.
Half-positional determinacy of infinite games.

Uri Zwick and Mike Paterson.
The complexity of mean payoff games on graphs.