Abstract—This paper describes the implementation and validation of a model predictive controller (MPC) for a photobioreactor system. The microalgae concentration, the solution pH and the illumination index, which is related to the lit PBR volume, are simultaneously controlled at imposed setpoints values by manipulating the dilution rate, the flow of CO$_2$ supplied to the culture and the incident light. The complex, time consuming nonlinear optimization is replaced by iterative quadratic programming optimizations. With a reduced number of tuning parameters and no demand for dynamics decoupling, the proposed controller improves considerably the speed of the control loop compared to the results available in literature.

I. INTRODUCTION

Cultures of microalgae have an interesting potential for the production of renewable energy. Their diversity accounts for a wide range of biofuels, i.e., biodiesel, biohydrogen, bioethanol, biogas. With lower requirements than other feedstocks, cultures of microalgae can reach higher yields in terms of fuel potential, and have lead to the concept of third-generation biofuels [1]. Microalgae can also be used to produce added-value compounds, such as long chain polyunsaturated fatty acids, neutral lipids, proteins, carbohydrates, pigments, vitamins etc. [2], [3]. Another major advantage of microalgae is their ability to use a variety of carbon sources for growth, and even more importantly, their potential to sustain photoautotrophic metabolism, using light as energy source to fix CO$_2$. In a context where CO$_2$ emissions from human activities are increasing, microalgae are solid candidates to achieve CO$_2$ mitigation. Thus, the importance of microalgae cultivation resides in their capacity to be redesigned as cell factories that require only solar light and inorganic substrates (i.e. CO$_2$, ammonia and phosphorus salts) to produce biofuels and added value compounds for food, feed, chemical and pharmaceutical sectors [4].

Nevertheless, reliable monitoring and control tools are required to sustainably develop microalgae industry towards achieving stable performance and process traceability [5]. To this end, the physico-chemical and biological phenomena involved in the process have to be translated into reliable mathematical models. In particular, the light-driven growth processes are described using a particular class of models that express the specific growth rate as a function of local irradianc. The radiative models, which express the light attenuation inside a culture of microalgae, are fundamental in photobioreactor studies [6]. The coupling between radiative models and growth kinetics is an active research topic and several dynamic models have been published [7], [8], [9], [10], [11].

Whereas the accurate modeling of microalgae cultivation has its elements of difficulty, the control of these processes is even more challenging. In addition to the typical nonlinear and complex characteristics of most biotechnological processes, specific limitations such as the heterogenous nature of the culture in terms of light distribution make the problem even more delicate to tackle. Recent results in the area of control of microalgae cultures include [12], [13], [14], [15], [16]. The biotechnological processes, and particularly the microalgae cultivation processes, are naturally multivariable and presents strong interactions between I/O channels requiring advanced control techniques to adapt to the dynamic changes in the structure of the system [17].

In this context, this study deals with the implementation and validation of a model predictive control (MPC) for a photobioreactor (PBR) system. MPC is particularly suited for controlling multivariable processes as it naturally considers the interconnections between the system inputs and outputs. The system outputs, i.e. biomass concentration, pH and the illumination index $\gamma$ (the ratio between the lighted volume and the dark volume), are controlled by manipulating the dilution rate, the influent flow rate of carbon dioxide and the incident light intensity. In view of a future implementation on a real PBR system, a MPC algorithm is developed which approximates the complex time-consuming optimization by successive quadratic optimization. Compared to other control strategies implemented on the same model, the proposed solution improves considerably the response time.

The paper is organized as follows. Section II presents the mathematical model which describes the PBR system, while Section III introduces the MPC algorithm used to control the system. Simulation results are presented and discussed in Section IV and conclusions are drawn in Section V.

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II. MODEL DESCRIPTION

The mathematical model, which describes the growth of the microalgae in the photobioreactor consists of three parts [10]: the biological model, the radiative-transfer model and the thermodynamic model. The biological model describes the evolution of the system states \( X, c_{TIN}, c_{TIC}, c_{O_2}, y_{CO_2}^{out}\) and \( y_{O_2}^{out}\), which respectively represent the biomass concentration, total inorganic nitrogen concentration, total inorganic carbon concentration, dissolved oxygen concentration, molar fraction of carbon dioxide in the output gas and molar fraction of oxygen in the output gas:

\[
\frac{dX}{dt} = r_x - DX
\]

\[
\frac{dc_{TIN}}{dt} = - \frac{Y_{N/X}}{M_x} r_x + D(c_{TIN,in} - c_{TIN})
\]

\[
\frac{dc_{TIC}}{dt} = - \frac{1}{M_x} r_x + D(c_{TIC,in} - c_{TIC}) + (K_{La})_{O_2} \frac{D_{CO_2}}{D_{O_2}} \left( \frac{(y_{O_2}^{in} + y_{O_2}^{out})/2 \cdot P}{\gamma_{CO_2} H_{CO_2}} - c_{CO_2} \right)
\]

\[
\frac{dc_{O_2}}{dt} = \frac{Q_p}{M_x} - Dc_{O_2} + (K_{La})_{O_2} \frac{D_{CO_2}}{D_{O_2}} \left( \frac{(y_{O_2}^{in} + y_{O_2}^{out})/2 \cdot P}{H_{O_2}} - c_{O_2} \right)
\]

\[
\frac{dy_{CO_2}^{in}}{dt} = \frac{RT}{P \cdot V_g} \left( \frac{(y_{CO_2}^{in} G_{in} - y_{CO_2}^{out} G_{out} - V_L (K_{La})_{O_2}}{D_{CO_2}} \right) \left( \frac{(y_{O_2}^{in} + y_{O_2}^{out})/2 \cdot P}{\gamma_{CO_2} H_{CO_2}} - c_{CO_2} \right)
\]

\[
\frac{dy_{CO_2}^{out}}{dt} = \frac{RT}{P \cdot V_g} \left( \frac{(y_{CO_2}^{in} G_{in} - y_{CO_2}^{out} G_{out} - V_L (K_{La})_{O_2}}{D_{CO_2}} \right) \left( \frac{(y_{O_2}^{in} + y_{O_2}^{out})/2 \cdot P}{H_{O_2}} - c_{O_2} \right)
\]

In (1)-(6), \( D \) is the dilution rate, \( r_x \) is the photosynthetic growth rate, \( c_{TIN,in} \) and \( c_{TIC,in} \) respectively represent total inorganic nitrogen and total inorganic carbon concentrations in the influent, \( y_{CO_2}^{in} \) and \( y_{O_2}^{in} \) are respectively the molar fractions of carbon dioxide and oxygen at inlet; \( Y_{N/X} \) is yield of total inorganic nitrogen (TIN) conversion, \( Q_p \) is the photosynthetic quotient, \( M_x \) is the C-mole mass; \( (K_{La})_{O_2} \) represents the overall volumetric mass-transfer coefficient for oxygen, \( D_{CO_2} \) and \( D_{O_2} \) respectively represent the molecular diffusivity of carbon dioxide and of oxygen; \( P \) is the total pressure, \( R \) is the universal gas constant and \( T \) is the temperature; \( H_{CO_2} \) and \( H_{O_2} \) respectively represent the Henry’s constant for \( CO_2 \) and for \( O_2 \); \( V_L \) and \( V_g \) are respectively the volume of liquid and of gas. \( G_{in} \) and \( G_{out} \) are the flows of gas at inlet and outlet, computed as

\[
G_{in} = G_{CO_2}^{in} + G_{in}^{O_2} + G_{in}^{N_2}
\]

\[
G_{out} = G_{CO_2}^{out} + G_{out}^{O_2} + G_{out}^{N_2}
\]

Based on (7), (8), the gas fractions in the inlet and outlet are respectively given by

\[
y_{in}^{CO_2} = \frac{G_{in}^{CO_2}}{G_{in}^{CO_2} + G_{in}^{O_2} + G_{in}^{N_2}}
\]

\[
y_{out}^{CO_2} = \frac{G_{out}^{CO_2}}{G_{out}^{CO_2} + G_{out}^{O_2} + G_{out}^{N_2}}
\]

Notice that nitrogen is neither produced nor consumed, thus its inlet flow rate equals its outlet flow rate. Hence

\[
y_{in}^{N_2} \cdot G_{in} = y_{out}^{N_2} \cdot G_{out}
\]

while (7), (9) and (8), (10) respectively imply that

\[
y_{in}^{CO_2} + y_{in}^{O_2} + y_{in}^{N_2} = 1
\]

\[
y_{out}^{CO_2} + y_{out}^{O_2} + y_{out}^{N_2} = 1
\]

The photosynthetic growth rate is highly dependent on the light availability inside the culture (\( G \)). Its attenuation along the PBR’s depth (\( z \)) is expressed by a radiative-transfer model [7] coupled with a kinetic law, which allows to compute local photosynthetic responses based on the photon flux density \( q_0 \). The integration of the local photosynthetic responses provides an average photosynthetic response over the PBR’s volume. Hence the photosynthetic growth rate is given by:

\[
r_x = \frac{\mu_0}{L} \int_0^L \mu(G(z)) dz \cdot f(pH) \cdot X - \mu_s \cdot X
\]

where

\[
\mu(G(z)) = \frac{G(z)}{K_i + G(z)}
\]

\[
q_0 \exp \left( -\frac{1 + \alpha}{2\alpha} E_a X \right)
\]

\[
f(pH) = \frac{(pH_{max} - pH)(pH_{max} - pH_{min})}{(pH_{max} - pH_{min})} \exp \left[ 1 - \frac{(pH_{max} - pH)(pH_{max} - pH_{min})}{(pH_{max} - pH_{min})} \right]
\]

Using (15), (16) in (14), the photosynthetic growth rate can be rewritten as

\[
r_x = \frac{\mu_0}{L} \cdot \frac{2\alpha}{(1 + \alpha)E_a X} \ln \left( \frac{K_i + q_0}{K_i + q_0 \exp \left( -\frac{1 + \alpha}{2\alpha} E_a X L \right)} \right)
\]

\[
f(pH) \cdot X - \mu_s X
\]

\[
L, \mu_0, \mu_s, K_i, \alpha = \sqrt{E_a/(E_a + 2bE_a)} \] is the linear scattering modulus, \( E_a \) and \( E_s \) are respectively the mass absorption and mass scattering coefficients and \( b \) is the backward scattering fraction. The illumination index, defined as the ratio between the light zone and the total volume of the reactor [18] is given by:

\[
\gamma = \frac{\ln \left( \frac{G_c}{q_0} \right)}{-\frac{1 + \alpha}{2\alpha} E_a X \cdot L}
\]

where \( G_c \) represents the irradiance at compensation point (where photosynthesis equals respiration).

The thermodynamic model refers to the \( NH_4 - CO_2 - H_2O \) multisolute system, which quantitatively describes the molecular and ionic species, as well as the ionic activities. The interacting chemical species in the culture broth are: carbon dioxide
(c\textsubscript{CO\textsubscript{2}}), bicarbonate ions (c\textsubscript{HCO\textsubscript{3}^-}), carbonate ions(c\textsubscript{CO\textsubscript{3}^-}), ammonia(c\textsubscript{NH\textsubscript{3}}), ammonium ions (c\textsubscript{NH\textsubscript{4}+}), carbohydrates ions (c\textsubscript{NH\textsubscript{2}COO^-}), hydroxyl ions (cOH^-) and hydrogen ions (cH^+). Hence, the mass balance of the species leads to

\begin{align*}
\text{cTIN} &= c\text{NH}_3 + c\text{NH}_4^+ + c\text{NH}_2\text{COO}^- \quad (20) \\
\text{cTIC} &= c\text{CO}_2 + c\text{HCO}_3^- + c\text{CO}_3^- + c\text{NH}_2\text{COO}^- \quad (21)
\end{align*}

Since the chemical reactions are much faster than the overall system dynamics, chemical equilibrium is assumed. The relationships between the species at chemical equilibrium are given in terms of species activities (products between activity coefficients and species concentrations):

\begin{align*}
K_1 &= \frac{\gamma\text{H}^+\text{CH}^- \cdot \gamma\text{HCO}_3^- \cdot \gamma\text{CO}_3^-}{\gamma\text{H}^+\text{CO}_2^2} \\
K_2 &= \frac{\gamma\text{H}^+\text{CH}^- \cdot \gamma\text{CO}_2^2 \cdot \gamma\text{CO}_3^-}{\gamma\text{HCO}_3^- \cdot \gamma\text{CO}_3^-} \\
K_3 &= \frac{\gamma\text{NH}_4^+ \cdot \gamma\text{NH}_3 \cdot \gamma\text{OH}^- \cdot \gamma\text{CO}_3^-}{\gamma\text{NH}_2\text{COO}^-} \\
K_4 &= \frac{\gamma\text{NH}_3 \cdot \gamma\text{NH}_2\text{COO}^-}{\gamma\text{H}^+ \cdot \gamma\text{CO}_3^-} \\
K_W &= \frac{\gamma\text{H}^+ \cdot \gamma\text{OH}^- \cdot \gamma\text{CO}_2^2}{\gamma\text{HCO}_3^- \cdot \gamma\text{CO}_3^-} \quad (22)
\end{align*}

where \(K_i, i = 1, 2, 3, 4, W\) are the equilibrium constants. The charge balance equation and the calculation of the solution pH complete the thermodynamical model:

\begin{align*}
\gamma\text{NH}_3 + \gamma\text{H}^+ + \gamma\text{CO}^2+ &= \gamma\text{HCO}_3^- + 2\gamma\text{CO}_3^- + \gamma\text{NH}_2\text{COO}^- + \gamma\text{OH}^- + \gamma\text{Cl}^- \quad (23) \\
pH &= -\log_{10}(\gamma\text{H}^+ \gamma\text{H}^+) \quad (24)
\end{align*}

The numerical values of the system parameters are given in Table I.

### Table I. Numerical Values of the Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B_a)</td>
<td>172m\textsuperscript{2}/Kg</td>
</tr>
<tr>
<td>(E_a)</td>
<td>870m\textsuperscript{2}/Kg</td>
</tr>
<tr>
<td>(b)</td>
<td>0.0008</td>
</tr>
<tr>
<td>(\mu_0)</td>
<td>0.16h\textsuperscript{-1}</td>
</tr>
<tr>
<td>(K_1)</td>
<td>120\textsuperscript{2}/m\textsuperscript{2}/s</td>
</tr>
<tr>
<td>(\mu_s)</td>
<td>0.013h\textsuperscript{-1}</td>
</tr>
<tr>
<td>(Y_{N/I})</td>
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</tr>
<tr>
<td>(M_I)</td>
<td>24.72e-3Kg/C-mol</td>
</tr>
<tr>
<td>(Q_p)</td>
<td>1.107</td>
</tr>
<tr>
<td>((K_La)_{O2})</td>
<td>0.9h\textsuperscript{-1}</td>
</tr>
<tr>
<td>(D_{CO2})</td>
<td>1.96e-9m\textsuperscript{2}/s</td>
</tr>
<tr>
<td>(D_{O2})</td>
<td>2.5e-9m\textsuperscript{2}/s</td>
</tr>
<tr>
<td>(P)</td>
<td>1.0135Pa</td>
</tr>
<tr>
<td>(R)</td>
<td>8.3143/mol/K</td>
</tr>
<tr>
<td>(T)</td>
<td>293.15K</td>
</tr>
<tr>
<td>(H_{CO2})</td>
<td>290.38Pa/m\textsuperscript{3}/mol</td>
</tr>
<tr>
<td>(H_{O2})</td>
<td>8.385e4Pa/m\textsuperscript{3}/mol</td>
</tr>
<tr>
<td>(G_c)</td>
<td>5\mu m\textsuperscript{2}/mol photons/s</td>
</tr>
</tbody>
</table>

### III. Multivariable Predictive Control

The successful use of MPC in advanced industrial applications lies in its ability to take into account constraints imposed on both process inputs (manipulated variables) and outputs (controlled variables), constraints which usually affect quality, economic efficiency and safety. Additionally, MPC techniques allow to address in a direct way the multivariable nature of the processes, the extension from single input single output to multiple inputs multiple outputs (MIMO) systems being straightforward. In this section we describe a MPC algorithm to control the biomass concentration (X), the pH of the solution (pH) and the illumination index (\(\gamma\)) of the PBR presented in Section II by manipulating the dilution rate (D), the inlet carbon dioxide gas flow rate (\(G_{\text{CO2}}\)) and the photon flux density (\(q_0\)). Although it would be possible to assign for each controlled output a manipulated input and implement a non-cooperative predictive control algorithm, we present here the cooperative predictive control implementation, in which the set of inputs (\(D, G_{\text{CO2}}^i, q_0\)) is computed based on the total control error.

All predictive control algorithms are based on few common key elements: a system model to compute predictions of the controlled system outputs, the online optimization to find the optimal control variables and the feedback mechanism to compensate for disturbances affecting the system. The structure of the control loop employed here is shown in Fig. 1. The objective of a model predictive controller is to find the future process input sequence that optimizes a cost function over the prediction horizon \(N_p\), taking the constraints on the process inputs and outputs into account. The simplest and most common cost index considers the summed squares of the predicted output deviations from the setpoint, although other terms such as penalties on the manipulated variables can also be included. The cost index considered here is

\[
J = \sum_{k=1}^{N_p} \left( X_{ref}(t + k) - X(t + k) \right)^2 + \sum_{k=1}^{N_p} \left( p_{H\text{ref}}(t + k) - p_{H}(t + k) \right)^2 + \sum_{k=1}^{N_p} \left( \gamma_{ref}(t + k) - \gamma(t + k) \right)^2
\]
where \( X^{ref}(t + k/t) \), \( pH^{ref}(t + k/t) \) and \( \gamma^{ref}(t + k/t) \), \( k = 1 \ldots N_p \), denote the prediction of the respective setpoints made at time instant \( t \), while \( X(t + k/t) \), \( pH(t + k/t) \) and \( \gamma(t + k/t) \), \( k = 1 \ldots N_p \), denote the prediction of the respective process outputs made at time instant \( t \). Since the process outputs are predicted using the model described in Section II, the minimization of the cost index \( J \) with respect to the process inputs \( D, G^{CO}_{in}, q_0 \) subjected to the constraints \( D_{min} \leq D \leq D_{max}, G^{CO}_{in,min} \leq G^{CO}_{in} \leq G^{CO}_{in,max}, q_{0,min} \leq q_0 \leq q_{0,max} \) results in a complex nonlinear optimization problem. The solution of this problem is difficult to get in due time in view of real-life implementation. Therefore we use here the algorithm described in [19], which replaces the complex, time consuming nonlinear optimization with iterative quadratic optimization problems, whose solutions converge to the nonlinear optimal solution.

Briefly, it is assumed that the future sequence of manipulated variables is the sum of a basic future control scenario and optimizing future control actions:

\[
D(t + k/t) = D_{base}(t + k/t) + \delta D(t + k/t) \\
G^{CO}_{in}(t + k/t) = G^{CO}_{in,base}(t + k/t) + \delta G^{CO}_{in}(t + k/t) \\
q_0(t + k/t) = q_{0,base}(t + k/t) + \delta q_0(t + k/t)
\]

(26)

\( k = 0 \ldots N_p - 1 \), which provide a first approximation of the output predictions as the cumulative result of the two effects:

\[
X(t + k/t) = \bar{X}(t + k/t) + X_{opt}(t + k/t) \\
pH(t + k/t) = \bar{pH}(t + k/t) + pH_{opt}(t + k/t) \\
\gamma(t + k/t) = \bar{\gamma}(t + k/t) + \gamma_{opt}(t + k/t)
\]

(27)

The response components \( \bar{X}(t + k/t) \), \( \bar{pH}(t + k/t) \) and \( \bar{\gamma}(t + k/t) \), \( k = 1 \ldots N_p \) are computed by applying the known sequences \( D_{base}(t + k/t), G^{CO}_{in,base}(t + k/t), q_{0,base}(t + k/t), k = 0 \ldots N_p - 1 \) to the model inputs, while the response components \( X_{opt}(t + k/t), pH_{opt}(t + k/t), \gamma_{opt}(t + k/t) \) are given by the optimizing sequences \( \delta D(t + k/t), \delta G^{CO}_{in}(t + k/t), \delta q_0(t + k/t) \). Assuming the optimizing components of the control actions inputs are small enough, it is possible to obtain an expression for \( X_{opt}(t + k/t), pH_{opt}(t + k/t), \gamma_{opt}(t + k/t) \) by linearizing the model along the trajectories described by \( D_{base}(t + k/t), G^{CO}_{in,base}(t + k/t), q_{0,base}(t + k/t) \). Hence, in matrix notations the prediction equations become

\[
\begin{align*}
X &= \bar{X} + G_{11} \cdot \delta D + G_{12} \cdot \delta G^{CO}_{in} + G_{13} \cdot \delta q_0 \\
pH &= \bar{pH} + G_{21} \cdot \delta D + G_{22} \cdot \delta G^{CO}_{in} + G_{23} \cdot \delta q_0 \\
\gamma &= \bar{\gamma} + G_{31} \cdot \delta D + G_{32} \cdot \delta G^{CO}_{in} + G_{33} \cdot \delta q_0
\end{align*}
\]

(28)

with

\[
\begin{align*}
X &= \begin{bmatrix} X(t + 1/t) & \ldots & X(t + N_p/t) \end{bmatrix}' \\
pH &= \begin{bmatrix} pH(t + 1/t) & \ldots & pH(t + N_p/t) \end{bmatrix}' \\
\gamma &= \begin{bmatrix} \gamma(t + 1/t) & \ldots & \gamma(t + N_p/t) \end{bmatrix}' \\
\bar{X} &= \begin{bmatrix} \bar{X}(t + 1/t) & \ldots & \bar{X}(t + N_p/t) \end{bmatrix}' \\
pH &= \begin{bmatrix} pH(t + 1/t) & \ldots & pH(t + N_p/t) \end{bmatrix}'
\end{align*}
\]

\( \gamma = \begin{bmatrix} \gamma(t + 1/t) & \ldots & \gamma(t + N_p/t) \end{bmatrix}' \)

\( \delta D = \begin{bmatrix} \delta D(t/t) & \ldots & \delta D(t + N_p - 1/t) \end{bmatrix}' \)

\( \delta G^{CO}_{in} = \begin{bmatrix} \delta G^{CO}_{in}(t/t) & \ldots & \delta G^{CO}_{in}(t + N_p - 1/t) \end{bmatrix}' \)

\( \delta q_0 = \begin{bmatrix} \delta q_0(t/t) & \ldots & \delta q_0(t + N_p - 1/t) \end{bmatrix}' \)

(29)

\( G_{ij}, i, j \in \{1, 2, 3 \} \) are the matrices of step response coefficients (\( i \) denotes the output, \( j \) denotes the input). By denoting

\[
\begin{align*}
G_1 &= \begin{bmatrix} G_{11} & G_{12} & G_{13} \end{bmatrix} \\
G_2 &= \begin{bmatrix} G_{21} & G_{22} & G_{23} \end{bmatrix} \\
G_3 &= \begin{bmatrix} G_{31} & G_{32} & G_{33} \end{bmatrix} \\
U &= \begin{bmatrix} \delta D & \delta G^{CO}_{in} & \delta q_0 \end{bmatrix}'
\end{align*}
\]

(30)

the cost index (25) can be rewritten in the standard form

\[
J(U) = U'HU + 2f'U + c
\]

(31)

where

\[
H = G'_1G_1 + G'_2G_2 + G'_3G_3
\]

(32)

\[
f = -(G'_1(X^{ref} - \bar{X}^t) + G'_2(pH^{ref} - \bar{pH}) + G'_3(\gamma^{ref} - \bar{\gamma}))
\]

\[
c = (X^{ref} - \bar{X})'(X^{ref} - \bar{X}) + (pH^{ref} - \bar{pH})'(pH^{ref} - \bar{pH}) + (\gamma^{ref} - \bar{\gamma})'(\gamma^{ref} - \bar{\gamma})
\]

(33)

which can be solved using quadratic programming techniques. Since (27) are only approximations, the control sequences \( D(t + k/t) = D_{base}(t + k/t) + \delta D(t + k/t), G^{CO}_{in}(t + k/t) = G^{CO}_{in,base}(t + k/t) + \delta G^{CO}_{in}(t + k/t) \) and \( q_0(t + k/t) = q_{0,base}(t + k/t) + \delta q_0(t + k/t) \) are suboptimal. By redefining \( D_{base}(t + k/t) = D(t + k/t), G^{CO}_{in,base}(t + k/t) = G^{CO}_{in}(t + k/t) \) and recomputing iteratively the control law components by repeating the procedure at the same sampling instant until

\[
|\delta D(t + k/t)| \leq \epsilon_D \\
|\delta G^{CO}_{in}| \leq \epsilon_{G^{CO}_{in}} \\
|\delta q_0(t + k/t)| \leq \epsilon_{q_0}
\]

an accurate approximation of the optimal control can be obtained [20].

IV. SIMULATION RESULTS AND DISCUSSION

The model described in Section II has been implemented in Matlab as a system of 16 differential-algebraic equations: 6 ordinary differential equations for the PBR states, 2 algebraic equations related to the gaseous system and 8 algebraic equations associated with the chemical species in the culture. The predictive controller, also implemented in Matlab, computes at every sampling instant \( T_s = 1h \) new values for the process inputs, which are constrained as follows: \( D \in [0, 0.5]h^{-1}, G^{CO}_{in} \in [0, 0.1]mol/h, q_0 \in [0, 15000]\mu mol \text{ photon/m}^2/\text{s} \). A prediction horizon \( N_p = 5 \) samples has been employed. The predictions of the setpoints over the prediction horizon have been set equal to the setpoint value at the present time instant, i.e. \( X^{ref}(t + k/t) = X^{ref}(t), pH^{ref}(t + k/t) = pH^{ref}(t), \gamma^{ref}(t + k/t) = \gamma^{ref}(t) \). The bounds on the control increments
Fig. 2. Predictive control of the photobioreactor at optimal setpoint values

Fig. 3. Predictive control of the photobioreactor for several setpoint changes

have been defined as $\epsilon_D = 5 \times 10^{-4}$, $\epsilon_{G_{CO_2}} = 5 \times 10^{-5}$, $\epsilon_{q_0} = 4$. At each sampling instant, the optimization has been carried out until either conditions (33) were simultaneously fulfilled or a maximum of 30 QP-problems have been solved.

Figure 2 illustrates an experiment in which the setpoints for the controlled variables have been set to $X^{ref} = 0.81$Kg/m$^3$, $pH^{ref} = 7.5$ and $\gamma^{ref} = 1$, which represent an optimum situation as determined in [18]. Noticeably in Fig. 2 is that although the control of $pH$ and $\gamma$ can occur quite fast, a slight deterioration of the performance on these outputs is introduced by the controller to accelerate the control of the biomass concentration. This is a feature of cooperative predictive control approach, such as the one implemented here and it is not due to a badly tuned control loop. After 75h, all outputs have reached the imposed setpoints. Compared to the results published in [18] where $X$ and $\gamma$ reach their setpoints in 200h and 150h respectively, the proposed predictive controller improves considerably the speed of the loop. Figure 3 illustrates a second experiment which starts in the optimal conditions detailed before. In the first part of the experiment
a setpoint change is imposed on the biomass concentration while the $pH$ and $\gamma$ must be kept at the same setpoint values. In the second part of the experiment the setpoint values for biomass concentration and $\gamma$ change at the same time instant. In all imposed scenarios, the multivariable MPC controller recomputes the optimal inputs and an effective control of the PBR system is obtained without the need of re-tuning the controller. The setpoint values for all controlled variables can be set independently, as long as they are reachable.

V. CONCLUSIONS

In this paper the multivariable predictive control of a complex photobioreactor system has been presented. The control loop has been implemented and evaluated in simulation. The obtained results indicate that the proposed control loop leads to better performances compared to the results available in literature. Aside the performance improvement that one may expect when using advanced control techniques, the emphasis in this paper was on using an algorithm which can provide an accurate approximation of the optimal control with sufficiently low computational effort. This requirement is of utmost importance in view of the envisaged real-life implementation of the control loop.

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