I. Introduction and motivations

Despite the unquestionable success of Einstein’s General Relativity (GR) at the solar system scale, some important phenomenons, such as Dark Matter and Dark Energy (accounting together for almost 95% of the matter-energy content of the known universe), remains beyond our understanding. What is Dark Matter “made of”? Where does Dark Energy “come from”? Over the last decades, there were numerous attempts to solve this problem. One of them is to consider that the unruled phenomenons are due to unknown degrees of freedom (that can be interpreted as new particles or as a new component in the description of gravity) and the most simple candidate for these degrees of freedom is a scalar field (generically named ϕ).

The aim of this poster is to present some general features of scalar-tensor theories of gravity and some results of my recent research related to black holes surrounded by scalar fields.

II. No (scalar) Hair Theorem

Surrounding a black hole with a scalar field is a challenging problem. In general, the simplest models are disfavoured, as illustrated by the following result due to Bekenstein:

Theorem 1 (Bekenstein) Consider a stationary asymptotically flat black hole spacetime

Hypothesis 1: Consider a minimally coupled real scalar field:

\[ S = \int_M \left[ F(g_{\mu\nu}, \partial_\alpha g_{\mu\nu}, \ldots) + \nabla_\mu \phi \nabla^\mu \phi - V(\phi) \right] \sqrt{-g} \, d^4 x \]

Hypothesis 2: The scalar field share the space-time symmetries.

Hypothesis 3: (Energetic condition) φV′(φ) ≥ 0 ∀φ, with V′(φ) = dV/dφ, and φV″(φ) = 0 for some discrete values of φ, say φi.

The scalar field must be trivial (φ(x, t) = φi, ∀x \in \partial M) on the black hole exterior region.

Consequently, in such theories, black holes will remain the same as in GR. Trying to circumvent this result (or any generalisation) motivate the study of scalar fields non-minimally coupled to gravity and/or violating some of the space-time symmetries.

IV. From shift-symmetry to spontaneous scalarization

One interesting subclass of scalar-tensor theories is characterized by the following action:

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{1}{16 \pi G} R - \nabla_\mu \phi \nabla^\mu \phi + f(\phi) I(g) \right] \]

where a complex scalar field is non-minimally coupled to gravity via a geometrical invariant I(g).

In such a model, the space-time curvature will act as a source for the scalar field via the term I(g) and lead to non trivial scalar field configurations. This mechanism is known as curvature induced scalarization. The pattern of the solutions depend on the invariant and on the term f(ϕ) defining the coupling. The case of a scalar field coupled to the Gauss-Bonnet invariant ICGB = R2 - 4RµνRµν + 4RµνRνµ has been studied in the literature for several choices of f(φ), revealing different kind of behaviours. For our study, we have focussed on a coupling to ICGB of the form f(φ) = γ1 |φ| + γ2 |φ|2, where γ1, γ2 are constants.

The case γ2 = 0, in which the theory is invariant under a shift of the scalar field, is known as the “shift-symmetric” case. In this case, hairy black holes do exist for several values of γ1 ∈ [0, γ1max]. The case γ1 = 0 is known in the literature as the “spontaneously scalarized” case, since hairy black holes only exist for 0 < γ2 < γ2 ≤ γ2max. We have constructed black hole solutions in the “general case” (γ1, γ2 ≠ 0) and shown how one can get hairy black holes in this case. In particular, for a given value of γ1 ≠ 0, solutions exist for 0 ≤ γ2 < γ2max but different values of γ1 reveal different patterns, as illustrated on figure 1.

On this figure, one can see how the range of accessible values of γ2 is influenced by the presence of γ1 and how the phenomenon of spontaneous scalarization (black line) appear in the limit γ1 → 0. For really small values of γ1 (see the blue curve), two different branches of solutions appear. The first one, connected to the shift-symmetric solutions (γ2 = 0) exist for γ2 ∈ [0, γ2max] where it connect to a second branch existing only for γ2 ∈ [γ2e, γ2max]. In the limit γ1 → 0 the first branch (which exist only for small values of ϕ(0)) “disappear” while the second branch remains to give the spontaneously scalarized solutions.

V. Conclusions

The study of scalar tensor theories is a large subject due to the variety of possible interactions between the scalar field and the metric tensor which lead to very different kind of behaviours. In the present work, we have shown the connections between two theories where black holes can be dressed by a scalar field thanks to spacetime curvature.

VI. Perspectives

We hope to return on these solutions in the near future to address the question of their stability. Furthermore, we plan to extend our study to different kind of scalar-tensor interactions.

References