Reconciling Rationality and Stochasticity: Rich Behavioral Models in Two-Player Games

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July 24, 2016

GAMES 2016 - 5th World Congress of the Game Theory Society
The talk in one slide

Two traditional paradigms for agents in complex systems

- **Fully rational**
  - System = (multi-player) game

- **Fully stochastic**
  - System = large stochastic process

In some fields (e.g., computer science), need to go beyond: **rich behavioral models**

*Illustration: planning a journey in an uncertain environment*
Full paper available on arXiv [Ran16a]: abs/1603.05072
1. Rationality & stochasticity

2. Planning a journey in an uncertain environment

3. Synthesis of reliable reactive systems

4. Conclusion
Rationality & stochasticity

Planning a journey in an uncertain environment

Synthesis of reliable reactive systems

Conclusion
Rationality hypothesis

**Rational agents** [OR94]:

- clear personal objectives,
- aware of their alternatives,
- form sound expectations about any unknowns,
- choose their actions coherently (i.e., regarding some notion of optimality).

⇒ In the particular setting of zero-sum games: antagonistic interactions between the players.

↩ Well-founded abstraction in computer science. E.g., processes competing for access to a shared resource.
Stochasticity

Stochastic agents:
- often a *sufficient abstraction* to reason about macroscopic properties of a complex system,
- agents follow stochastic models that can be based on experimental data (e.g., traffic in a town).

Several models of interest:
- fully stochastic agents $\implies$ Markov chain [Put94],
- rational agent against stochastic agent $\implies$ *Markov decision process* [Put94],
- two rational agents $+$ one stochastic agent $\implies$ stochastic game or competitive MDP [FV97].
Choosing the appropriate paradigm matters!

As an agent having to choose a strategy, the assumptions made on the other agents are crucial.

$$\Rightarrow$$ They define our objective hence the adequate strategy.

$$\Rightarrow$$ Illustration: planning a journey.
1. Rationality & stochasticity

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Aim of this illustration

Flavor of ≠ types of **useful strategies** in stochastic environments.

- Based on a series of papers, most in a computer science setting (more on that later) [Ran13, BFRR14b, BFRR14a, RRS15a, RRS15b, BCH+16].

Applications to the **shortest path problem**.

→ Find a **path of minimal length** in a weighted graph (Dijkstra, Bellman-Ford, etc) [CGR96].
Aim of this illustration

Flavor of $\neq$ types of **useful strategies** in stochastic environments.

- Based on a series of papers, most in a computer science setting (more on that later) [Ran13, BFRR14b, BFRR14a, RRS15a, RRS15b, BCH+16].

Applications to the **shortest path problem**.

What if the environment is **uncertain**? E.g., in case of heavy traffic, some roads may be crowded.
Planning a journey in an uncertain environment

Each action takes time, target = work.

▷ What kind of strategies are we looking for when the environment is stochastic (MDP)?
Solution 1: minimize the expected time to work

“Average” performance: meaningful when you journey often.

Simple strategies suffice: no memory, no randomness.

Taking the car is optimal: $\mathbb{E}_D(\text{TS}^\text{work}) = 33$. 
Solution 2: traveling without taking too many risks

Minimizing the *expected time* to destination makes sense if we travel often and it is not a problem to be late.

With car, in 10% of the cases, the journey takes 71 minutes.
Solution 2: traveling without taking too many risks

Most bosses will not be happy if we are late too often...

what if we are risk-averse and want to avoid that?
Solution 2: maximize the *probability* to be on time

**Specification:** reach work within 40 minutes with 0.95 probability

**Sample strategy:** take the train \( \sim \mathbb{P}_D[TS^{\text{work}} \leq 40] = 0.99 \)

**Bad choices:** car (0.9) and bike (0.0)
Solution 3: strict worst-case guarantees

**Specification:** guarantee that work is reached within 60 minutes (to avoid missing an important meeting)

**Sample strategy:** bike \(\leadsto\) worst-case reaching time = 45 minutes.

**Bad choices:** train \((wc = \infty)\) and car \((wc = 71)\)
Solution 3: strict worst-case guarantees

Worst-case analysis $\sim$ two-player zero-sum game against a rational antagonistic adversary (bad guy)

- forget about probabilities and give the choice of transitions to the adversary
Solution 4: minimize the *expected* time under strict worst-case guarantees

- **Expected time**: car $\sim E = 33$ but $wc = 71 > 60$
- **Worst-case**: bike $\sim wc = 45 < 60$ but $E = 45 >>> 33$
Solution 4: minimize the *expected* time under strict worst-case guarantees

In practice, we want both! Can we do better?

▷ **Beyond worst-case synthesis** [BFRR14b, BFRR14a]: minimize the expected time under the worst-case constraint.
Solution 4: minimize the *expected* time under strict worst-case guarantees

Sample strategy: try train up to 3 delays then switch to bike.

\[ wc = 58 < 60 \text{ and } E \approx 37.34 << 45 \]

\[ \Rightarrow \text{Strategies need memory} \Rightarrow \text{more complex!} \]
Solution 5: multiple objectives $\Rightarrow$ trade-offs

Two-dimensional weights on actions: *time* and *cost*.

Often necessary to consider *trade-offs*: e.g., between the probability to reach work in due time and the risks of an expensive journey.
Solution 5: multiple objectives $\Rightarrow$ trade-offs

Solution 2 (probability) can only ensure a single constraint.

- **C1**: 80% of runs reach work in at most 40 minutes.
  - Taxi $\sim \leq 10$ minutes with probability $0.99 > 0.8$.

- **C2**: 50% of them cost at most $10$ to reach work.
  - Bus $\sim \geq 70\%$ of the runs reach work for $3$.

Taxi $\not\models C2$, bus $\not\models C1$. What if we want $C1 \land C2$?
Solution 5: multiple objectives ⇒ trade-offs

- **C1**: 80% of runs reach work in at most 40 minutes.
- **C2**: 50% of them cost at most 10$ to reach work.

Study of **multi-constraint percentile queries** [RRS15a].

- Sample strategy: bus once, then taxi. Requires *memory*.
- Another strategy: bus with probability 3/5, taxi with probability 2/5. Requires *randomness*.
Solution 5: multiple objectives ⇒ trade-offs

- **C1**: 80% of runs reach work in at most 40 minutes.
- **C2**: 50% of them cost at most 10$ to reach work.

Study of multi-constraint percentile queries [RRS15a].

In general, both memory and randomness are required.

≠ previous problems ⇨ more complex!
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Controller synthesis

- **Setting:**
  - a reactive **system** to control,
  - an **interacting environment**,  
  - a **specification** to enforce.

- For **critical** systems (e.g., airplane controller, power plants, ABS), testing is not enough!
  ⇒ Need **formal methods**.

- **Automated synthesis** of provably-correct and efficient controllers:
  - mathematical frameworks,
    e.g., games on graphs [GTW02, Ran13, Ran14]
  - software tools.
Strategy synthesis in stochastic environments

**Strategy** = formal model of how to control the system

1. How complex is it to decide if a winning strategy exists?
2. How complex such a strategy needs to be? **Simpler is better.**
3. Can we synthesize one efficiently?

⇒ Depends on the winning objective, the exact type of interaction, etc.
Some other objectives

The example was about **shortest path objectives**, but there are many more! Some examples based on energy applications.

- **Energy**: operate with a (bounded) fuel tank and never run out of fuel \([BFL^{+}08]\).
- **Mean-payoff**: average cost/reward (or energy consumption) per action in the long run \([EM79]\).
- **Average-energy**: energy objective + optimize the long-run average amount of fuel in the tank \([BMR^{+}15]\).

Also inspired by economics:

- **Discounted sum**: simulates interest or inflation \([BCF^{+}13]\).
Conclusion

Our research aims at:

- defining meaningful *strategy concepts*,
- providing *algorithms* and *tools* to compute those strategies,
- classifying the *complexity* of the different problems from a theoretical standpoint.

→ Is it mathematically possible to obtain efficient algorithms?

Take-home message

Rich behavioral models are natural and important in computer science (e.g., synthesis).

Maybe they can be useful in other areas too. E.g., in economics: combining sufficient risk-avoidance and profitable expected return, value-at-risk models.

Thank you! Any question?
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Algorithmic complexity: hierarchy of problems

For shortest path

Solutions 2 (P) and 5 (percentile)

Solution 4 (BWC)

Solutions 1 (P) and 3 (wc)

UNDECIDABLE
not computable by an algorithm

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