Algebraic structures in linear dynamics

Karl Grosse-Erdmann
Université de Mons
Belgium

Many operators that are commonly studied in linear dynamics are defined on spaces that have an additional multiplicative structure. For example, the translation operator $T : f \to f(\,\cdot\, + 1)$ and the differentiation operator D admit dense orbits when considered on the space $H(\mathbb{C})$ of all entire functions. Now, one may multiply entire functions, and this turns $H(\mathbb{C})$ in a natural way into a Fréchet algebra. So one may wonder if there exists a subalgebra of $H(\mathbb{C})$ that consists entirely, apart form 0, of vectors that have a dense orbit under T, or under D; such algebras are called hypercyclic algebras. The answer is negative for T (Aron et al. 2007) and positive for D (Shkarin, Bayart-Matheron, 2010). We report here on recent work by several authors on the question of the existence of hypercyclic algebras for more general operators.