Case $F \in q^-$. By symmetry, it suffices to show $\mathbf{r}_a \models q_{[x \mapsto b]}$ and $\mathbf{r}_a \not\models q_{[x \mapsto a]}$. The result $\mathbf{r}_a \models q_{[x \mapsto b]}$ follows from $\Theta_b(q^+) \subseteq \mathbf{r}_a$ and $\Theta_b(F) \notin \mathbf{r}_a$.

We next show $\mathbf{r}_a \not\models q_{[x \mapsto a]}$. Assume, towards a contradiction, a valuation μ over $\text{vars}(q) \setminus \{x\}$ such that $\mathbf{r}_a \models \mu(q_{[x \mapsto a]})$, thus $\mu(F_{[x \mapsto a]}) \notin \mathbf{r}_a$. Define $\mu_{[x \mapsto a]} := \mu \cup \{x \mapsto a\}$, a valuation over vars($q$) which is obviously distinct from Θ_b. Since $\mu_{[x \mapsto a]}(q^+) \subseteq \mathbf{r}_a$, it follows, by our previous reasoning, that either $\mu_{[x \mapsto a]} = \Theta_a$ or $\mu_{[x \mapsto a]} = \Theta_b$. So it must be the case that $\mu_{[x \mapsto a]} = \Theta_a$. We obtain $\mu(F_{[x \mapsto a]}) = \mu_{[x \mapsto a]}(F) = \Theta_a(F) \in \mathbf{r}_a$, a contradiction.

Case $F \in q^+$. By symmetry, it suffices to show $\mathbf{r}_a \models q_{[x \mapsto a]}$ and $\mathbf{r}_a \not\models q_{[x \mapsto b]}$. It is straightforward that $\mathbf{r}_a \models q_{[x \mapsto a]}$.

We next show $\mathbf{r}_a \not\models q_{[x \mapsto b]}$. Assume, towards a contradiction, a valuation μ over $\text{vars}(q) \setminus \{x\}$ such that $\mathbf{r}_a \models \mu(q_{[x \mapsto b]})$, thus $\mu(F_{[x \mapsto b]}) \in \mathbf{r}_a$. Define $\mu_{[x \mapsto b]} := \mu \cup \{x \mapsto b\}$, a valuation over vars($q$) which is obviously distinct from Θ_a. Since $\mu_{[x \mapsto b]}(q^+) \subseteq \mathbf{r}_a$, it follows, by our previous reasoning, that either $\mu_{[x \mapsto b]} = \Theta_a$ or $\mu_{[x \mapsto b]} = \Theta_b$. So it must be the case that $\mu_{[x \mapsto b]} = \Theta_b$. We obtain $\mu(F_{[x \mapsto b]}) = \mu_{[x \mapsto b]}(F) = \Theta_b(F) \in \mathbf{r}_a$, a contradiction. □

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