1 Introduction

Let $\Omega \subseteq \mathbb{R}^n$ be open bounded and $2 < p < 2^*$, where $2^* = \frac{2n}{n-2}$ for $N > 2$ and $2^* = +\infty$ if $N = 2$.

Lane-Emden problem:

$$\Delta u + u^p = 0, \quad \text{in} \quad \Omega,$$
$$u = 0, \quad \text{on} \quad \partial \Omega.$$

Solutions are critical points of energy functional

$$J(u) = \frac{1}{p+1} \int_{\Omega} |\nabla u|^{p+1} - \frac{1}{p} \int_{\Omega} |u|^{p+1},$$

where $H^1_0(\Omega) = H^1_0(\Omega)$.

2. Compute $E$.

3. Compute $g$.

1. Let $\varphi_\epsilon : \mathbb{R} \to [0,1]$, smooth, compactly supported

Sketch: There exists a ground-state solution.

3 Schrödinger problem

$$-\Delta u + V(x)u = |u|^{p-2}u, \quad \text{in} \quad \mathbb{R}^n,$$
$$u = 0, \quad \text{on} \quad \partial \Omega.$$

Schrodinger problem:

$$-\Delta u + V(x)u = |u|^{p-2}u, \quad \text{in} \quad \Omega,$$
$$u = 0, \quad \text{on} \quad \partial \Omega.$$

with $V$ continuous, bounded, $-\Delta + V$ self-adjoint and $0$ does not belong to the spectrum $\sigma(-\Delta + V)$.

Solutions are critical points of energy functional

$$J(u) = \frac{1}{p+1} \int_{\Omega} |\nabla u|^{p+1} - \frac{1}{p} \int_{\Omega} |u|^{p+1}.$$

2 Lane-Emden problem

Theorem 1: (Mountain-Pass theorem, [1] and [3])

There exist a non-trivial one-signed solution and a sign-changing solution.

Sketch: They are minimum of $\varphi_\epsilon$ on $\mathcal{M}_\delta := \{u \neq 0 : \varphi_\epsilon'(u) = 0\}$ and $\mathcal{M}_\epsilon := \{u : u^2 \leq \epsilon\}$.

Mountain-Pass Algorithm: ([4] and [9])

1. Let $u \in H^1_0(\Omega)$ and $n \rightarrow 0$.
2. Compute $u_0 := P(u)$.
3. Compute $g := -\Delta u_0$.
4. If $\|V\|_{\mathcal{M}_\delta} \leq 1$ stop; else $v := P(u_0 - g)$.
5. If $\varphi_\epsilon(v) < \varphi_\epsilon(u_0)$, go to step 3 ;
6. $\varphi_\epsilon \rightarrow \infty$ and go to step 4 ;
where the projection $P$ equals $P_0 : H^1_0 \rightarrow \mathcal{M}_\epsilon$.
Algorithm: ([7])

$$\min \{u \in S(\omega) : \varphi(u) = 0\}$$

where $u \in S(\omega)$.

We work with projection $P : H^1_0 \rightarrow \mathcal{M}_\epsilon$ which associates at $u$ the maximum of energy in $H^1_0(u)$.

3 Schrödinger problem

Theorem 2: ([8])

There exists a ground-state solution.

Sketch: It is minimum of $\varphi_\epsilon$ on $\mathcal{M}_\delta := \{u \neq 0 : \varphi_\epsilon' (u) = 0\}$ and $\mathcal{M}_\epsilon := \{u : u^2 \leq \epsilon\}$.

Algorithm: ([7]) Same idea as before:

$$\min \{u \in S(\omega) : \varphi(u) = 0\}$$

where $u \in S(\omega)$.

References