A walk through scalar tensor gravity

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Plan

Plan

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- Context
- No hair theorem(s)

2 Horndeski

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4 Status after GW&GRB170817

5 Conclusions & Outlooks

Introduction : *Why* should we modify general relativity ?

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Despite consequential success . . .

- Offer a geometrical explanation of gravitational process [elegant]
- Allow to explain many phenomenons :
  1. Mercury perihelion problem
  2. Existence and shape of gravitational waves : GW150914 (2016)
  3. Gravitational lensing : Event Horizon telescope (2019)

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. . . there are unexplained phenomena within General Relativity (GR) :

- Origin and value of the cosmological constant
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Not all of them reduces to quantum correction problems !

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One of them is to consider that the unrated phenomena are due to unknown degrees of freedom (that can be interpreted as new particles or as a new component in the description of gravity).

In GR, all the degrees of freedom are encoded in the metric $g_{\mu\nu}$.
But, formally, the equivalence principle does not rule out the possible existence of other kind of fields in the model.

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- Important element of many models :
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- Also experimentally motivated since the Brout-Englert-Higgs boson's discovery (CERN 2012)

Introduction : Why not considering the simplest case ?

Why not just using $\mathscr{L}_{EKG} = \kappa\left(R - 2\Lambda\right) - \frac{1}{2}\nabla_{\mu}\phi\nabla^{\mu}\phi - V(\phi)$?

Introduction : Why not considering the simplest case ?

No Scalar-Hair Theorem (*Schematically*)

Consider an asymptotically flat black hole spacetime

Hypothesis 1 : (Symmetries of spacetime)

Hypothesis 2 : (Symmetries of the scalar field)

Hypothesis 3 : (Coupling condition)

Hypothesis 4 : (Energetic condition)

Then, *the scalar field must be trivial :* $\phi\left(x^{\mu}\right) = c^{te}, \forall x^{\mu}$.

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Ex : Consider a minimally coupled real scalar field :

$$S = \int_{\mathcal{M}} \left[F(g_{\mu\nu}, \partial_\alpha g_{\mu\nu}, \dots) - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) \right] \sqrt{-g} \, \mathrm{d}^n x$$

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Then, *the scalar field must be trivial :* $\phi\left(x^{\mu}\right)=c^{te}, \forall x^{\mu}$.

Note : Generically, the proof makes **no use** of the Einstein's equations. It just uses the scalar field equation defined thanks to hypothesis 3.

No Scalar-Hair Theorem (*Schematically*)

Consider an asymptotically flat black hole spacetime

Sketch of proof (for the example).

□

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The spacetime is stationnary

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Under the first hypothesis[1], Hawking's *rigidity theorem* establish that the spacetime must be either static or axisymetric.

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If the spacetime is axisymetric, it pocesses one time-like Killing vector $\vec{m}$ and one space-like Killing vector with closed orbits $\vec{k}$.

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If the spacetime is axisymetric, it pocesses one time-like Killing vector $\vec{m}$ and one space-like Killing vector with closed orbits $\vec{k}$.
Using a coordinate system (t, r, θ, φ) adapted to these isometries, one simply have $\vec{m} = \partial_t$ and $\vec{k} = \partial_\varphi$.
The metric will then be such that $\partial_t g_{\mu\nu} = 0 = \partial_\varphi g_{\mu\nu}$.

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[1] and assuming the null energy condition

No Scalar-Hair Theorem (*Schematically*)

Consider an asymptotically flat black hole spacetime

Hypothesis 2 : (Symmetries of the scalar field)
The scalar field shares the space-time symmetries.

Sketch of proof (for the example).

From hypothesis 2, the scalar field will then also be such that $\partial_t \phi = 0 = \partial_\varphi \phi$.

□

No Scalar-Hair Theorem (*Schematically*)

Consider an asymptotically flat black hole spacetime

Hypothesis 3 : (Coupling condition)
Consider a minimally coupled real scalar field :

$$S = \int_{\mathcal{M}} \left[F(g_{\mu\nu}, \partial_\alpha g_{\mu\nu}, \dots) - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) \right] \sqrt{-g} \, \mathrm{d}^n x$$

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Hypothesis 3 fixes that the scalar field must satisfy the Klein-Gordon equation (where $\approx$ denotes an on-shell equality)

$$\nabla_\mu \nabla^\mu \phi - V'(\phi) \approx 0.$$

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Hypothesis 3 fixes that the scalar field must satisfy the Klein-Gordon equation (where $\approx$ denotes an on-shell equality)

$$\nabla_\mu \nabla^\mu \phi - V'(\phi) \approx 0.$$

Multiplying the above equation by ϕ and integrating over the black-hole exterior spacetime region $\mathfrak{E}$, one get

$$\int_{\mathfrak{E}} \left(\phi \nabla^\mu \nabla_\mu \phi - \phi V'(\phi)\right) \sqrt{-g}\, \mathrm{d}^4 x \approx 0.$$

The core of the proof will then be to work with this equality to obtain a positively defined intergand that will constrain the scalar field to be trivial.

No Scalar-Hair Theorem (*Schematically*)

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Performing an intergration by part over the first term gives

$$\begin{aligned} 0 &\approx \int_{\mathfrak{E}} \left(\phi \nabla^{\mu} \nabla_{\mu} \phi - \phi V'(\phi)\right) \sqrt{-g}\, \mathrm{d}^4 x \\ &= -\int_{\mathfrak{E}} \left(\nabla^{\mu} \phi \nabla_{\mu} \phi + \phi V'(\phi)\right) \sqrt{-g}\, \mathrm{d}^4 x + \int_{\mathfrak{E}} \nabla^{\mu} \left(\phi \nabla_{\mu} \phi\right) \sqrt{-g}\, \mathrm{d}^4 x \\ &= -\int_{\mathfrak{E}} \left(\nabla_{\mu} \phi \nabla^{\mu} \phi + \phi V'(\phi)\right) \sqrt{-g}\, \mathrm{d}^4 x + \int_{\partial \mathfrak{E}} \left(\phi \nabla_{\mu} \phi\right) \sqrt{-g}\, n^{\mu}\, \mathrm{d}^3 \sigma. \end{aligned}$$

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Performing an intergration by part over the first term gives

$$\begin{aligned}0 &\approx \int_{\mathfrak{E}} \left(\phi\nabla^{\mu}\nabla_{\mu}\phi - \phi V'\left(\phi\right)\right)\sqrt{-g}\ \mathrm{d}^4x \\ &= -\int_{\mathfrak{E}} \left(\nabla^{\mu}\phi\nabla_{\mu}\phi + \phi V'\left(\phi\right)\right)\sqrt{-g}\ \mathrm{d}^4x + \int_{\mathfrak{E}} \nabla^{\mu}\left(\phi\nabla_{\mu}\phi\right)\sqrt{-g}\ \mathrm{d}^4x \\ &= -\int_{\mathfrak{E}} \left(\nabla_{\mu}\phi\nabla^{\mu}\phi + \phi V'\left(\phi\right)\right)\sqrt{-g}\ \mathrm{d}^4x + \int_{\partial\mathfrak{E}} \left(\phi\nabla_{\mu}\phi\right)\sqrt{-g}\ n^{\mu}\ \mathrm{d}^3\sigma.\end{aligned}$$

We will now, first, argue that the boundary term actually vanishes.

□

No Scalar-Hair Theorem (*Schematically*)

Consider an asymptotically flat black hole spacetime

Sketch of proof (for the example).

Since $\mathfrak{E}$ correspond to the black hole exterior spacetime, $\partial\mathfrak{E}$ consist of the event horizon $\mathcal{H}$ and the spacetime asymptotic region $\mathcal{H}_\infty$ such that

$$\int_{\partial\mathfrak{E}} \phi\nabla_\mu\phi\sqrt{-g}\, n^\mu \, \mathrm{d}^3\sigma = \int_{\mathcal{H}} \phi\nabla_\mu\phi\sqrt{-g}\, n^\mu \, \mathrm{d}^3\sigma + \int_{\mathcal{H}_\infty} \phi\nabla_\mu\phi\sqrt{-g}\, n^\mu \, \mathrm{d}^3\sigma$$

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The term computed on $\mathcal{H}_\infty$ vanishes since the spacetime is asymptotically flat (and then we should have $\nabla_\mu\phi \to 0$ «sufficently fast» when approaching $\mathcal{H}_\infty$).

□

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The term computed on $\mathcal{H}$ also vanishes :

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The term computed on $\mathcal{H}$ also vanishes :
Since the event horizon of a stationnary, asymptotically flat blak hole is a *Killing horizon*, n^μ, the normal to $\mathcal{H}$, will be a linear combination of the Killing fields ∂_t and ∂_φ.

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The term computed on $\mathcal{H}$ also vanishes :
Since the event horizon of a stationnary, asymptotically flat blak hole is a *Killing horizon*, n^μ, the normal to $\mathcal{H}$, will be a linear combination of the Killing fields ∂_t and ∂_φ.
Now, since $\partial_t\phi = 0 = \partial_\varphi\phi$ (hyp. 2), we get that $(n^\mu\nabla_\mu\phi)\,|_{\mathcal{H}} = 0$, which ensure that the term vanishes[1].

□

[1] Provided ϕ and $\mathrm{d}^3\sigma$ are both finite at the horizon.

No Scalar-Hair Theorem (*Schematically*)

Consider an asymptotically flat black hole spacetime

Sketch of proof (for the example).

With this at hand, our previous equation

$$-\int_{\mathfrak{E}} \left(\nabla_\mu \phi \nabla^\mu \phi + \phi V'(\phi)\right) \sqrt{-g}\ \mathrm{d}^4 x + \int_{\partial \mathfrak{E}} \left(\phi \nabla_\mu \phi\right) \sqrt{-g}\ n^\mu\ \mathrm{d}^3 \sigma \approx 0.$$

reduces to

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We now want to justify that each term in the integrant is a positive quantity over $\mathfrak{E}$.

□

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$$\int_{\mathfrak{E}} \left(\nabla_\mu\phi\nabla^\mu\phi + \phi V'(\phi)\right)\sqrt{-g}\,\mathrm{d}^4x \approx 0.$$

First, $\nabla_\mu\phi\nabla^\mu\phi \geq 0$.

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$$\int_{\mathfrak{E}} \left(\nabla_\mu \phi \nabla^\mu \phi + \phi V'(\phi)\right) \sqrt{-g}\, \mathrm{d}^4 x \approx 0.$$

First, $\nabla_\mu \phi \nabla^\mu \phi \geq 0$.
Indeed, since ϕ shares the spacetime symetries (hyp. 2), it must be invariant under both Killing fields ∂_t and ∂_φ.
As a consequences, its gradient must be orthogonal to these vectors and will then be either spacelike or null; that is $\nabla_\mu \phi \nabla^\mu \phi \geq 0$.

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Hypothesis 4 : (Energetic condition)
$\phi V'(\phi) \geq 0 \;\; \forall \phi$, with $V'(\phi) = \mathrm{d}V/\mathrm{d}\phi$, & $\phi V'(\phi) = 0$ for some discrete values of ϕ, say ϕ_i.

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To be able to conclude now, we will rely on hypothesis 4 which will constrain the form of the potential to ensure that one always have :

$$\int_{\mathfrak{E}} \left(\nabla_\mu \phi \nabla^\mu \phi + \phi V'(\phi)\right) \sqrt{-g}\, \mathrm{d}^4 x \geq 0.$$

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***Then**, the scalar field must be trivial :* $\phi(x^\mu) = c^{te}, \forall x^\mu$ *on the black hole exterior region.*

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$$\int_{\mathfrak{E}} \left(\nabla_\mu\phi\nabla^\mu\phi + \phi V'(\phi)\right)\sqrt{-g}\,\mathrm{d}^4x \geq 0.$$

The only way to saturate the lower bound of this inequality is then to have $\phi(x^\mu) = c^{te}$ everywhere on $\mathfrak{E}$. □

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$$S = \int_{\mathcal{M}} \mathscr{L} \sqrt{-g}\, \mathrm{d}^4 x$$

Horndeski

$$\begin{aligned}\mathscr{L} =& K(\phi,\rho) - G_3(\phi,\rho)\Box\phi + G_4(\phi,\rho)R + G_{4,\rho}(\phi,\rho)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ &+ G_5(\phi,\rho)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ &- \frac{1}{6}G_{5,\rho}(\phi,\rho)\left[(\Box\phi)^3 - 3\Box\phi\left(\nabla_\mu\nabla_\nu\phi\right)^2 + 2\left(\nabla_\mu\nabla_\nu\phi\right)^3\right],\end{aligned}$$

where

$$\rho = \nabla_\mu\phi\nabla^\mu\phi,$$

and where the functions $G_i(\phi,\rho)$ $(i \in \{3,4,5\})$ & $K(\phi,\rho)$ are **arbitrary** functions.

Horndeski

Examples

$$\begin{aligned}\mathscr{L} =& K(\phi,\rho) - G_3(\phi,\rho)\Box\phi + G_4(\phi,\rho)R + G_{4,\rho}(\phi,\rho)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ &+ G_5(\phi,\rho)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ &- \frac{1}{6}G_{5,\rho}(\phi,\rho)\left[(\Box\phi)^3 - 3\Box\phi\left(\nabla_\mu\nabla_\nu\phi\right)^2 + 2\left(\nabla_\mu\nabla_\nu\phi\right)^3\right],\end{aligned}$$

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With $G_i = 0, \forall i \in \{3,4,5\}$, you get

$$\mathscr{L} = K(\phi,\rho)$$

Horndeski

Examples

$$\mathscr{L} = K(\phi,\rho) - G_3(\phi,\rho)\Box\phi + G_4(\phi,\rho)R + G_{4,\rho}(\phi,\rho)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi,\rho)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5,\rho}(\phi,\rho)\left[(\Box\phi)^3 - 3\Box\phi\left(\nabla_\mu\nabla_\nu\phi\right)^2 + 2\left(\nabla_\mu\nabla_\nu\phi\right)^3\right],$$

With $G_i = 0, \forall i \in \{3,4,5\}$, you get

$$\mathscr{L} = K(\phi,\rho)$$

and choosing $K(\phi,\rho) = -\frac{1}{2}\rho - V(\phi)$,

$$\mathscr{L}_{KG} = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)$$

Horndeski

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$$\begin{aligned}\mathscr{L} =& K(\phi,\rho) - G_3(\phi,\rho)\Box\phi + G_4(\phi,\rho)R + G_{4,\rho}(\phi,\rho)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ &+ G_5(\phi,\rho)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ &- \frac{1}{6}G_{5,\rho}(\phi,\rho)\left[(\Box\phi)^3 - 3\Box\phi\left(\nabla_\mu\nabla_\nu\phi\right)^2 + 2\left(\nabla_\mu\nabla_\nu\phi\right)^3\right],\end{aligned}$$

With $G_3 = 0 = G_5, G_4 = \kappa = c^4/16\pi\mathscr{G}$ and $K = -2\kappa\Lambda = c^{te}$, we obtain

$$\mathscr{L}_{EH} = \kappa\left(R - 2\Lambda\right)$$

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While $K(\phi,\rho) = -\frac{1}{2}\rho - V(\phi) - 2\kappa\Lambda$ give

$$\mathscr{L}_{EKG} = \kappa\left(R - 2\Lambda\right) - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)$$

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If $G_i(\phi,\rho) = G_i(\rho), \forall i \in \{3,4,5\}$ and $K(\phi,\rho) = K(\rho)$, the system posses an invariance under $\phi \to \phi + c^{te}$ (shift-symmetry) and the EOM for ϕ reduce to a conservation law (Nœther) :

$$\nabla_\mu J^\mu = 0.$$

Horndeski

Construction : schematically

Let us briefly discuss the steps in the discovery/construction of this lagrangian density from the (more recent) point of view of Galileon theory.

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   As a proof of concept let us consider the following toy model : $\tilde{\mathscr{L}} = \frac{1}{2}\phi\, \Box\phi - V(\phi) = \frac{1}{2}\phi\ \partial_\mu\partial_\nu\phi\ \eta^{\mu\nu} - V(\phi)$.

Horndeski

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$$\left(\frac{\partial}{\partial\phi} - \partial_\mu\left[\frac{\partial}{\partial(\partial_\mu\phi)}\right] + \partial_\mu\partial_\nu\left[\frac{\partial}{\partial(\partial_\mu\partial_\nu\phi)}\right]\right)\tilde{\mathscr{L}} = 0$$

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so we get

$$\left(\frac{1}{2}\Box\phi - V'(\phi) - \partial_\mu \left[0\right] + \partial_\mu \partial_\nu \left[\frac{1}{2}\phi \; \eta^{\mu\nu} \right] \right) = 0$$

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so we get

$$\left(\frac{1}{2}\Box\phi - V'(\phi) - \partial_\mu\,[0] + \partial_\mu\partial_\nu\left[\frac{1}{2}\phi\,\eta^{\mu\nu}\right]\right) = 0$$

$$\Leftrightarrow \;\frac{1}{2}\Box\phi - V'(\phi) + \frac{1}{2}\eta^{\mu\nu}\,\partial_\mu\partial_\nu\phi = \Box\phi - V'(\phi) = 0.$$

This becomes obvious if we realize that

$$\tilde{\mathscr{L}} = \frac{1}{2}\phi\,\partial_\mu\partial_\nu\phi\,\eta^{\mu\nu} - V(\phi) = -\frac{1}{2}\partial_\mu\phi\,\partial_\nu\phi\,\eta^{\mu\nu} + \partial_\mu\left(\frac{1}{2}\phi\,\partial_\nu\phi\,\eta^{\mu\nu}\right) - V(\phi)$$

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1. Figure out how one can avoid higher order derivatives in the EEL for a lagrangian density polynomial in the $\partial_\mu \partial_\nu \phi$'s.

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   Here again, to illustrate the mechanics of the argument let us consider an other toy model :

$$\bar{\mathscr{L}} = \mathcal{T}^{\mu_1 \mu_2 \nu_1 \nu_2} \, \partial_{\mu_1} \partial_{\nu_1} \phi \, \partial_{\mu_2} \partial_{\nu_2} \phi,$$

   where $\mathcal{T} = \mathcal{T}(\phi)$.

Horndeski

Construction : schematically

$$\bar{\mathscr{L}} = \mathcal{T}^{\mu_1\mu_2\nu_1\nu_2}\ \partial_{\mu_1}\partial_{\nu_1}\phi\ \partial_{\mu_2}\partial_{\nu_2}\phi$$

Horndeski

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$$\bar{\mathscr{L}} = \mathcal{T}^{\mu_1\mu_2\nu_1\nu_2}\ \partial_{\mu_1}\partial_{\nu_1}\phi\ \partial_{\mu_2}\partial_{\nu_2}\phi$$

Here again, the Euler-Lagrange equations (EEL) are given by

$$\left(\frac{\partial}{\partial\phi} - \partial_\mu\left[\frac{\partial}{\partial(\partial_\mu\phi)}\right] + \partial_\mu\partial_\nu\left[\frac{\partial}{\partial(\partial_\mu\partial_\nu\phi)}\right]\right)\bar{\mathscr{L}} = 0$$

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- $\dfrac{\partial\bar{\mathscr{L}}}{\partial\phi} = \partial_\phi\mathcal{T}^{\mu_1\mu_2\nu_1\nu_2}\ \partial_{\mu_1}\partial_{\nu_1}\phi\ \partial_{\mu_2}\partial_{\nu_2}\phi \leftarrow$ OK

Horndeski

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One can then avoid higher order terms by imposing some (anti-)symmetry condition on $\mathcal{T}$. For example, since **partial derivatives commute**, we could impose $\mathcal{T}^{\mu\nu\rho\sigma} = -\mathcal{T}^{\nu\mu\rho\sigma}$ and $\mathcal{T}^{\mu\nu\rho\sigma} = -\mathcal{T}^{\mu\nu\sigma\rho}$

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Construction : schematically

Let us briefly discuss the steps in the discovery/construction of this lagrangian density from the (more recent) point of view of Galileon theory.

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→ First, consider the study of a scalar field on flat spacetime (fixed Minkowski background) :

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1. Avoid higher order derivatives in EEL for polynomials in $\partial_\mu \partial_\nu \phi$'s.
2. Carefully construct the most general expression satisfying the condition.

One then get the lagrangian density for the (generalized) Galileon.

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2. Since **covariant derivatives does not commute**, this lead to higher order EEL. One should then kill them by adding the (unique) appropriate counter terms built with the curvature tensor.

One then get the lagrangian density for the covariant (generalized) Galileon.

Horndeski

Construction : schematically

Horndeski had “cracked” the problem from a completely different sarting point
(He directly asked the question of the most general lagrangian density [in $4D$] presenting at most second order equations for $g_{\mu\nu}$ and ϕ)

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↑ ↑ ↑ ↑ ↑

This result is non-trivial.

Even though the generalized Galileon provided the most general lagrangian density with second order field equation for ϕ on flat spacetime there was *a priori* no reasons why its covariant extension should still be the most general possibility on curved spacetime !

Horndeski

Has a final note on this construction, let us come back to the Horndeski lagrangian density and emphasize the link between the different terms.

Especially, let us emphasize which terms necessitate the introduction of an approrpiated counter term

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\begin{aligned}
\mathscr{L} =& K(\phi,\rho) - G_3(\phi,\rho)\Box\phi + G_4(\phi,\rho)R + G_{4,\rho}(\phi,\rho)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\
& + G_5(\phi,\rho)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\
& - \frac{1}{6}G_{5,\rho}(\phi,\rho)\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right],
\end{aligned}
$$

where

$$
\rho = \nabla_\mu\phi\nabla^\mu\phi,
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and where the functions $G_i(\phi,\rho)$ $(i \in \{3,4,5\})$ & $K(\phi,\rho)$ are **arbitrary** functions.

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&+ \underline{G_5(\phi,\rho)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi} \\
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To ~~infinity~~ Horndeski . . . and beyond : Motivation

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$$\text{Beyond Horndeski} \Longrightarrow \text{Higher order equations}$$

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→ Motivation : Applying some invertible disformal transformations

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = C(\rho, \phi) g_{\mu\nu} + D(\rho, \phi) \nabla_\mu \phi \nabla_\nu \phi,$$

one can map Horndeski theory to higher order theories but invertible field redefinition does not change the number of degrees of freedom.

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one can map Horndeski theory to higher order theories <u>but</u> invertible field redefinition does not change the number of degrees of freedom.
$\Longrightarrow$ there must be higher order theories propagating a single scalar degree of freedom (*i.e.* free of [Ostrodgradski's] ghost).

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The key to construct these "healthy" higher order theories is to rely on some <u>degeneracy condition</u> for the kinetic matrix of the theory

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The key to construct these “healthy” higher order theories is to rely on some degeneracy condition for the kinetic matrix of the theory (In agreement with Ostrogradski’s theorem which takes the non-degeneracy of the kinetic matrix as an hypothesis).

→ To perfom this analysis, one should rely on a $3+1$-slicing of spacetime.

To ~~infinity~~ Horndeski . . . and beyond : DHOST theories

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→ This then leads to the so called Degenerate Higher Order Scalar Tensor (DHOST) theories.

- DHOST theories have been fully classified up to cubic order in $\nabla_\mu \nabla_\nu \phi$
- The explicit form of the degeneracy condition allow to identify different sectors in the DHOST theories.

DHOST theories : a (very [very!]) quick overview

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→ This **does not imply** either that that all these DHOST theories are automatically free from the gradient instability.
  - Simply, the question then reduces to the corresponding one in Horndeski gravity (with possibly some exotic coupling to matter).

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→ For Horndeski gravity, the general formula for the propagation of GW is given by

$$c_{\mathsf{GW}}^2 = \frac{G_4 - \rho(\ddot{\phi}G_{5,\rho} + G_{5,\phi})}{G_4 - 2\rho G_{4,\rho} - \rho(H\dot{\phi}G_{5,\rho} - G_{5,\phi})}c^2$$

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→ If one requires $c_{\mathsf{GW}} = c$ irrespective of the background cosmological evolution, this then imposes that $G_{4,\rho} = 0 \wedge G_5 = 0$ which only leaves the following part of Horndeski theory :

$$\mathscr{L} = K(\phi,\rho) - G_3(\phi,\rho)\Box\phi + G_4(\phi)R$$

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So it might be (according to the RA) that alternative theories of gravity with $c_{\mathsf{GW}} \neq c$ can still provide some viable low energy description of the UV completion of GR but then one would need a mechanism explaining why/how $c_{\mathsf{GW}}(\lambda) \underset{\lambda \to \lambda_{\mathsf{obs}}}{\longrightarrow} c$.

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The road to interesting/healthy/fruitful alternative theories of gravity is sinous and paved by stringent constraints and no-go results.
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The road to interesting/healthy/fruitful alternative theories of gravity is sinous and paved by stringent constraints and no-go results.
The construction of these theories then requires to call into question the hypothesis of these results.

- → What is the influence of the non-minimal coupling to matter suggested by DHOST theories ?
- → Can one construct "beyond Horndeski like" theories outside of the metric formulation of GR ? (ex : teleparallel gravity)

Thank you for your attention !

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