EXTENDED SHELL MODEL CALCULATION
OF THE T=0 NATURAL PARITY SPECTRUM OF $^4$He

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The ground state and the first excited state of $^4$He are calculated in a complete 10$\hbar \omega$ shell model basis using the potentials of Volkov and Brink and Boeker. The $1^-$, $2^+$ and $4^+$ $T=0$ states are also investigated, respectively, up to 9, 8 and 6$\hbar \omega$. It is found that the agreement with experiment is significantly improved when the model space is enlarged.

The shell model is built on the idea that the low-energy nuclear properties can be described by using an appropriate effective interaction among particles moving in an average field. In practice this field is mostly approximated by a harmonic oscillator well and, moreover, only the excitations of particles from the ground state configuration to the lowest unoccupied orbits are included in the calculations. Of course, the configurations with higher excitation energy may have important effects, but they are generally ignored merely because their treatment leads to great technical difficulties. For very light nuclei, however, it is possible to investigate their contribution thoroughly. In the present work we address this problem for $^4$He.

Many attempts have been made to obtain a satisfactory level scheme for this nucleus in the context of the harmonic oscillator shell model. Early works were restricted to 2$\hbar \omega$ model spaces. In more recent studies the configurations up to 4$\hbar \omega$ excitation energy have been included in the shell model bases [1,2]. These calculations improve the 2$\hbar \omega$ results to some extent but, nevertheless, they remain unsatisfactory in that the first 0$^-$ excited state is still predicted too high in energy if the constraint of predicting correctly the binding energy is imposed. Our aim is to show that a more substantial enlargement of the model space is profitable. Therefore we present results obtained in a shell model space extended up to 10$\hbar \omega$.

Our computing procedure was described in ref. [3]. It will be sufficient to state that the operator matrices are constructed in four-nucleon harmonic oscillator bases including all the configurations coupled to zero total angular momentum and isospin up to $N_0$ harmonic oscillator quanta with $N_0=0, 2, ..., 10$. Owing to the completeness of the bases with regard to the excitation energy, the eigenstates of the energy matrices constructed using a translationally invariant hamiltonian are automatically eigenstates of the center-of-mass (CM) hamiltonian with eigenvalues $E_{\text{cm}}$ equal to $(2n_{\text{cm}}+l_{\text{cm}}+\frac{1}{2})\hbar \omega$. In fact the eigenstates characterized by $n_{\text{cm}}=0$ in the $N_0\hbar \omega$ space are at the same time good states ($E_{\text{cm}}=\frac{1}{2}\hbar \omega$) in a basis coupled to angular momentum $J=l_{\text{cm}}$ and including all the natural parity configurations up to $N_0\hbar \omega$ oscillator quanta with $N=N_0-J$. In this way we were able to extract from the various energy matrices, besides the ground state and the lowest 0$^+$ excited state up to $N=10$, the lowest $1^-$, $2^+$ and $4^+ T=0$ states, respectively, up to $N=9, 8$ and 6. The charge radius has been calculated from the expression

$$R_{\text{ch}} = \sqrt{\frac{1}{4} \left< \phi_{\text{SM}} \left| \sum_{i=1}^{4} r_i^2 \right| \phi_{\text{SM}} \right> + R_N^2 - \frac{5}{3} b^2}^{1/2}.$$  

The nucleon radius $R_N=0.772$ fm is extracted from the nucleon form factor given by Janssens et al. [4]. The term depending on the oscillator length param-
Fig. 1. Absolute energies of the ground state (solid lines) and the first excited state (dashed lines) obtained in the various \( N\hbar \omega \) spaces as functions of the oscillator length parameter using the potentials \( V_7 \) and \( B_1 \).

The two lowest good eigenvalues \( E_\nu (\text{GS}) \) and \( E_\nu (0^-) \) of the energy matrices constructed in the various \( N\hbar \omega \) spaces are displayed in fig. 1 as functions of the oscillator length parameter. It is apparent that for \( b \) varying from 1.7 to 2.1 fm the potentials \( V_7 \) and \( B_1 \) yield quite similar results. The absolute energy \( E_\nu (0^-) \) of the first excited state is a slowly varying function of \( b \) with a minimum for \( b \) equal to about 2 fm and, in addition, this energy is strongly reduced when the model space is enlarged. As seen from table 1, in a 10\( \hbar \omega \) calculation \( E_\nu (0^-) \) drops to \(-7.2\) MeV for the potential \( V_7 \) and to \(-7.0\) MeV for the potential \( B_1 \), which should be compared with the experimental value of \(-8.2\) MeV. One is thus prompted to calculate the shell-model energy spectrum of \(^4\text{He}\) for \( b \) near 2 fm. The fact that the dependence of the ground state energy upon \( b \) is drastically weakened with increasing \( N \) favours this procedure. In fact, for \( N = 10 \) the difference between \( E_\nu (\text{GS}) \) and its Coulomb-corrected experimental counterpart \((-29\) MeV\) remains less than 1 MeV when \( b \) is varied between 1.7 and 2.1 fm. Likewise, the

<table>
<thead>
<tr>
<th>( V_7 )</th>
<th>( \text{GS} )</th>
<th>( 0^+ )</th>
<th>( 2^+ )</th>
<th>( N )</th>
<th>( 1^- )</th>
<th>( \text{B1} )</th>
<th>( \text{GS} )</th>
<th>( 0^- )</th>
<th>( 2^- )</th>
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<td>3.7</td>
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Table 2: Theoretical charge radii (in fm) of $^4$He obtained in the various $N\hbar\omega$ spaces for $b=1.7$ and 2.0 fm using the potentials V7 and B1.

<table>
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<tr>
<th>N</th>
<th>V7</th>
<th>B1</th>
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<tr>
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<tr>
<td>10</td>
<td>1.71</td>
<td>1.72</td>
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The dependence of the excitation energy $E_{x,N}=E_x(0^+)-E_N(GS)$ of the first excited state upon $b$ becomes very smooth when the model space is enlarged beyond $6\hbar\omega$. Moreover, $E_{x,N}$ calculated in the $10\hbar\omega$ space using $b=2$ fm is equal to 20.8 MeV for the potential V7 and 21.7 MeV for the potential B1, which is very encouraging if one considers that in existing shell-model calculations of $^4$He this energy is predicted several MeV higher in energy than the experimental value of 20.1 MeV.

The values of the oscillator length parameter resulting from the above analysis are markedly larger than those currently used in shell-model studies of $^4$He but, remarkably enough, in the $10\hbar\omega$ space they yield theoretical charge radii close to the experimental value of 1.68 fm [8], as seen from table 2 where the values of $R_{ch}$ calculated for $b$ equal to 1.7 and 2.0 fm are displayed as functions of $N$.

The absolute energies $E_N(1^-)$ and $E_N(2^-)$ of the lowest $1^-$ and $2^+ T=0$ states present the same characteristics as $E_N(0^+)$. In particular, these energies are minimal for $b$ equal to 2 fm. As seen from table 1, for this value of $b$ our $8\hbar\omega$ calculations predict the lowest $2^+$ state at an excitation energy $E_8(2^-)-E_8(GS)$ equal to about 27 MeV. On the other hand, it is apparent that for small values of $N$ the position of the lowest $1^-$ state with regard to the positive parity spectrum cannot be defined properly because the differences between values of $E_N$ corresponding to adjacent values of $N$ are comparable to the separation of the excited states. In this respect we feel that the ambiguity inherent in comparing positive with negative parity states in small configuration spaces has not always been fully recognized. When the configuration space is enlarged sufficiently this difficulty disappears. For instance, comparing $E_7(1^-)$ and $E_8(1^-)$ with $E_8(GS)$ we obtain for the lowest $1^-$ state an excitation energy ranging from 23.3 MeV to 24.0 MeV for the potential V7 and from 23.8 MeV to 24.5 MeV for the potential B1.

In fact the $1^-$ and $2^- T=0$ states have given rise to controversy. In the survey on $^4$He done by Fiarman and Meyerhof [9] in 1973 these states are both considered as uncertain and have excitation energies equal, respectively, to 31 and 33 MeV. However, more recently Grüebler et al. [10] have reported overwhelming evidence for a $1^-$ state at 24.1 MeV. The existence of such a level at a comparable energy (24.4 MeV) results also from a $R$-matrix analysis of four-body reactions by Hale and Dodder [11]. The same analysis yields a $2^+$ level at 26.8 MeV. Our $8\hbar\omega$ results are thus compatible with the conclusions of these recent studies.

In their analysis of the $^2$$H(d,p)^3$$H$ reaction Grüebler et al. suggest also the possible existence of a $4^- T=0$ level at 24.6 MeV excitation energy. Such a level is neither in the survey by Fiarman and Meyerhof nor in the spectrum proposed by Hale and Dodder. On the other hand, a $4\hbar\omega$ calculation carried out by Bevelacqua [12] predicts the lowest $4^- T=0$ level at an excitation energy equal to 42.4 MeV and, in the present work, the excitation energy $E_8(4^-)-E_8(GS)$ obtained in the $6\hbar\omega$ space varies roughly from 42 to 36 MeV when $b$ is increased from 1.7 to 2.0 fm. It is worth noting that a calculation by Liu and Zamick [13] describing this level as a deformed four-particle–four-hole state leads to a similar energy range. The existence in the spectrum of $^4$He of a $4^+$ excitation at an energy comparable to the energies of the lowest excited state is thus questionable; more definite spin assignments are needed to clarify this question.
To summarize, our results suggest that a correct description of $^4$He in the context of the shell model requires the handling of very large model spaces. Such an effort appears essential, for instance, to solve the long-standing problem of bringing the energy of the first excited state in agreement with the experimental data when the constraint of predicting correctly the ground state properties is imposed. Our results demonstrate also that for the values of the oscillator length parameter considered in the present work the four-nucleon energy matrices constructed using two interactions which differ considerably from one another at short distance have comparable eigenvalues, even when configurations with excitation energies as large as $10\hbar\omega$ are included in the calculations. Actually, this result supports the assumption underlying implicitly most nuclear structure investigations, that a real understanding of the short-range correlations induced by the presence of a repulsive core in the two-particle effective potential is not required to describe nuclear spectra reliably. In this respect, the fact that our large-basis calculations yield energies for the $J^o = 1^-$ and $2^+ T=0$ states compatible with those resulting from recent experimental investigations is significant.

Of course further work is necessary to measure the capabilities of very large basis shell-model calculations. Effective interactions breaking the degeneracy of the $J^o = 0^-, 1^-, 2^+ T=0$ levels of $^4$He which belong to the same SU(4) supermultiplet should be utilized. Along these lines very preliminary results based on the Sussex matrix elements are encouraging. Moreover, it would be worth performing similar calculations for heavier nuclei. This is evidently a very difficult task. It must be realized that our calculations required the handling of operator matrices in a vector space comprising $2^{765}$ antisymmetrized states constructed so as to have good total angular momentum and isospin. Nevertheless, they were carried out on a modest computer (an IBM 4341) and, consequently, it is not unthinkable that using present-day computational facilities our computing procedure could be extended to the study of the lightest $p$-shell nuclei.

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References