Switching Control of An Activated Sludge Process

Eduardo Mojica-Nava[1], Christian Feudjio[2], and Alain Vande Wouwer[2]

***Abstract*— The activated sludge process is a complex microbial system used for biological wastewater treatment, i.e., carbon and ammonium removal. The main control objective is to regulate the outflow water quality despite disturbances in the inflow, such as influent flow, or influent concentration. The alternate phase process consists of a single bioreactor where oxygen is supplied intermittently to create nitrification and denitrification conditions while the biomass is recycled continuously from a settler. In this study, a switched affine model is first derived from an existing dynamic model. Then, a switching signal law and a feedback control are developed to reduce the ammonium concentration while keeping the closed-loop system stable. The proposed control strategy is validated in simulation with MATLAB/Simulink considering several influent scenarios such as dry, rainy, and stormy weather conditions.**

***Index Terms*— Switched affine systems, switching control, wastewater treatment.**

I. INTRODUCTION

The activated sludge process (ASP) is a complex microbial system used for biological wastewater treatment, i.e., carbon and ammonium removal [1]. The main control objective is to regulate the outflow water quality despite disturbances in the inflow, such as influent flow, or influent concentration [2]. 00C vvbbbbb jjuu8 losed-loop control strategies should be proposed to accomplish this operational objective. For several decades the control design of the ASP have been challenging [3]. Several traditional control techniques have been proposed for the reference model ASM1 of five reactor, focusing on different control objectives [4], [5]. Modern control strategies have been also proposed such as in [6], where a robust control algorithm is presented for maintaining the oxygen concentration in the aerobic tank. Recently, model predictive control methods have been presented mainly based on long-term simulations (see [7], [8], and references therein). Mostly, all of these works are based on the five tank benchmark simulation model No. 1 (BSM1). In contrast, in this paper we are dealing with a single reactor with two alternate phase operation modes. The alternate phase ASP is considered for carbon and nitrogen removal. The process consists of a single bioreactor where oxygen is supplied intermittently to create nitrification and denitrification conditions while the biomass is recycled continuously from a settler as illustrated in Fig. 1 [9].

A dynamic process model can be cast in the form of a switched affine model. In switched systems, a switching signal and an external control input may be designed together to guarantee closed-loop asymptotically stabilization [10], [11], [12]. In the particular case of switched linear systems, several effective techniques have been presented and summarized in books such as in [13], [10]. A relevant characteristic of the resulting affine system is the existence of a region of the state space including the equilibrium points due to the affine terms. In this case, we are trying to stabilize the system to this equilibrium set instead of convergence to zero [14], [15].

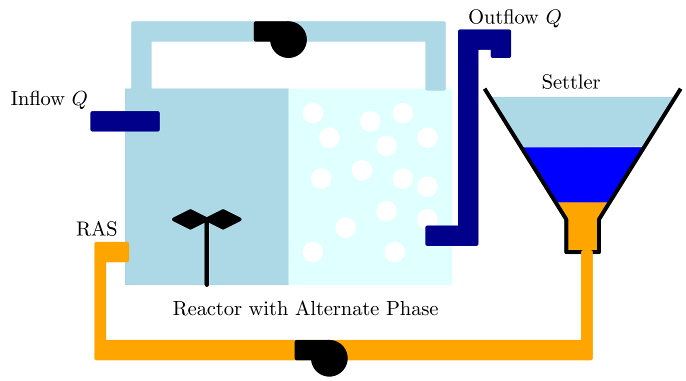

Fig. 1. Alternate Phase Activated Sludge Process

The main contribution of this paper is twofold. First, a switched affine model adapted from a two phase continuous activated sludge process is obtained for control design. Then, a switching signal law and an external feedback control are developed for the ASP model. Finally, the proposed control strategy is validated by simulation with a MATLAB/Simulink model.

The rest of the paper is organized as follows. Section II presents a brief description of the ASP as a switched affine system. In Section III, some theoretical results are shown for control design. Section IV presents simulation results and finally in Section V some conclusions are drawn.

II. ALTERNATE PHASE ACTIVATED SLUDGE PROCESS

Traditionally, nitrification and denitrification in activated sludge processes is the preferred biological process in water treatment plants. The main objective of the process is to regulate the outflow water quality despite disturbances in the inflow, such as influent flow, or influent concentration [2]. Closed-loop control strategies should be proposed to accomplish this operation objective. The alternate phase process consists of a single bioreactor where oxygen is supplied intermittently to create nitrification and denitrification conditions. This way, the aerobic and anoxic phases are accomplished intermittently in the same tank, and biomass is

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recycled continuously from a settler, which is assumed ideal [9].

Several models for the activated sludge process have been proposed since the reference model (ASM1) was presented by the I.A.W.Q. task group [16]. In this work, a reduced switched linear model with four variables is considered as suggested in [9]. It is based on a reduced nonlinear model [17], which is then transformed into a switched linear systems assuming that the Monod terms are replaced by linear terms [9], [2], [6]. This switched linear model is the starting point for control design. The values of coefficients and influent wastewater characteristics are taken from [9].

The switched linear model is initially described as

$$\dot{x}(t) = A_\sigma x(t) + B_\sigma u(t) \tag{1}$$

with matrices A_i and B_i as follows

$$A_1 = \begin{bmatrix} -(D_s + D_c) - 203.8 & 0 & 0 & 0 \\ 0 & -(D_s + D_c) & 26.73 & 0 \\ -10.92 & 0 & -26.73 - (D_s + D_c) & 0 \\ -67.25 & 0 & -122.2 & -k_L a - (D_s + D_c) \end{bmatrix} \tag{2}$$

$$B_1 = \begin{bmatrix} D_c & D_s & 0 & 2215.78 \\ 0 & 0 & 0 & 121.89 \\ 0 & 0 & D_s & -61.2 \\ 0 & 0 & 0 & -557.03 + 9.5 k_L a \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -(D_s + D_c) - 84.9 & 30.5 & 0 & 0 \\ -9.8 & -(D_s + D_c) - 2.17 & 0 & 0 \\ -4.55 & -1 & -(D_s + D_c) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

$$B_2 = \begin{bmatrix} D_c & D_s & 0 & 371.74 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & D_s & 60.7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where the state vector is $x = [S_S \; S_{NO_3} \; S_{NH_4} \; S_{O_2}]^\top$ with S_S the substrate concentration (in $g.m^{-3}$) in the bioreactor, S_{NO_3} the nitrate concentration, S_{NH_4} the ammonium concentration, and S_{O_2} the dissolved oxygen concentration. The input vector is $u = [S_{S_C} \; S_{Sin} \; S_{NH_4in} \; 1]^\top$, where S_{S_C} is the supplied carbon source concentration, S_{Sin} and S_{NH_4in} are the substrate and ammonium concentration in the influent, respectively. $\sigma \in \{1, 2\}$ are used to designate the switches between the aerobic phase (A_1, B_1) and the anoxic phase (A_2, B_2). In [9], a sensitivity analysis shows that the influence of the inflow substrate concentration S_{Sin} is moderate, whereas the influence of the inflow ammonium concentration S_{NH_4in} is critical. Hence, S_{Sin} can be considered as a constant known input, while S_{NH_4in} is considered as unknown input disturbance. The supplied external carbon S_{S_C} can be used as a continuous control variable. On the other hand, as oxygen is used only in phase 1 (aerobic phase), it can be considered as a continuous control variable for subsystem 1 only. In this work, the air transfer coefficient $k_L a$ is assumed to be a known constant value. Based on these considerations and assumptions, the switched linear model can be cast as a switched affine model, which is more suitable for control design

$$\dot{x} = A_\sigma x + \xi_\sigma + Bu \tag{4}$$
$$y = C_\sigma x$$

where ξ_σ is the affine vector, which is mainly based on the influent characteristics and is assumed constant as follows

$$\xi_1 = \begin{bmatrix} S_{Sin} D_s + 2215.78 \\ 121.9 \\ S_{NH_4in} D_s - 61.2 \\ -557.03 + 9.5 k_L a \end{bmatrix}$$

$$\xi_2 = \begin{bmatrix} S_{Sin} D_s + 371.74 \\ 0 \\ S_{NH_4in} D_s + 60.7 \\ 0 \end{bmatrix}$$

The new input vector is the same for both subsystems

$$B = \begin{bmatrix} D_c \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

and the continuous control variable $u = S_{S_C}$. The continuous process is performed at constant dilution rates, e.g., $D_s = 1.1433$ 1/d and $D_c = 0.016$ 1/d. The air transfer coefficient is also assumed constant $k_L a = 114$ 1/d. In the next section, the switched affine system (4) with matrices A_i and B is used to develop a control algorithm for both the switching signal and the continuous external input.

III. State Feedback Switching Control

In this section, we present a state feedback switching control algorithm for the activated sludge process modeled as a switched affine system (4). The main objective is to design a feedback stabilization control law, which involves finding appropriate switching signals as well as continuous state feedback controllers to make the closed-loop system stable and to follow a pre-defined reference for the soluble carbon. Recall that the system is single-input since we have

$$B = [D_c, 0, \ldots, 0]^\top,$$

which is the same for both subsystems. In this work, only the soluble carbon source is considered as a continuous control input for practical implementation reasons. However, an extension of the algorithm to two-input control variables could be achieved considering a pre-defined reference for the ammonium concentration. In addition, the affine vector changes the traditional stabilization problem into a stabilization problem that drives the states to some point of the equilibrium set X_ξ, such that $x(t) \to x_\xi$ as $t \to \infty$, for some $x_\xi \in X_\xi$. Then we have the definition of stabilizability of switched affine systems.

Definition 1: System (4) is said to be linear feedback stabilizable, if there exist a switching signal σ, and state feedback control inputs

$$u_i = -K_i(x - x_\xi) \tag{5}$$

where $x_\xi \in X_\xi$ is a constant reference input, such that the closed-loop switched system

$$\dot{x} = (A_\sigma - BK_\sigma)(x - x_\xi) + \xi_\sigma$$

is well-posed and uniformly asymptotically stable.

In order to develop a control algorithm to satisfy Definition 1, we apply the average technique to approximate the switching affine system. First, let us define the simplex set as

$$\Delta = \left\{ \alpha_i \in \mathbf{R} | \sum_{i=1}^{N} \alpha_i = 1, \alpha_i \geq 0 \right\}.$$

where N is the number of subsystems and the set of subsystems indexes is defined as $i \in \mathcal{I} = \{1, 2, \ldots, N\}$. The average matrices are defined as convex combinations of the subsystems matrices. We have the following assumptions of the average system [10], [11]

Assumption 2: There is a convex combination of A_i, $i \in \mathcal{I}$ which is Hurwitz.

The average matrix is then defined as

$$A_0 = \sum_{i \in \mathcal{I}} \alpha_i A_i$$

with $\alpha_i \in \Delta$. Also we set $B_0 = \alpha_1 B$, and the average affine vector as $\xi_0 = \sum_{i \in \mathcal{I}} \alpha_i \xi_i$. In this way, we obtain the average system [15]

$$\dot{x} = A_0 + \xi_0 + B_0 u, \tag{6}$$

which is used to design the control algorithm as follows. Based on the average system (6), we can obtain the equilibrium set X_ξ. The equilibrium set is a set of equilibrium points $x_\xi \in \mathbf{R}$ such that $\lim_{t \to \infty} x(t) = x_\xi$ holds for all initial conditions whenever the feedback law $u(t)$ and the switching signal $\sigma(t)$ are applied. To obtain x_ξ, assume that the external input $u(t) = 0$. Then, from (6) we obtain the equilibrium points $\dot{x} = 0$ as

$$A_0 x + \xi_0 = \sum_{i \in \mathcal{I}} \alpha_i (A_i + \xi_i) = 0,$$

which allow us to define the equilibrium set as follows

$$X_\xi = \{x \in \mathbf{R}^n : \sum_{i \in \mathcal{I}} \alpha_i (A_i + \xi_i) = 0, \alpha_i \in \Delta\}. \tag{7}$$

Notice that only equilibrium points $x_\xi \in X_\xi$ can be reached by the stabilization strategy. The controller design is based on the two following results that establish the quadratically stabilization conditions, which are based on results for switched affine systems [15].

Lemma 3: System (4) is quadratically stabilizable if there exist gain matrices K_i, $i \in \mathcal{I}$ such that the matrix

$$\left\{ \sum_{i \in \mathcal{I}} \alpha_i (A_i - BK_i) : \alpha_i \in \Delta \right\}$$

contains a Hurwitz matrix. In addition, it is necessary that there exist an equilibrium point $x_\xi \in X_\xi$ such that

$$\sum_{i \in \mathcal{I}} \alpha_i (A_i - BK_i) x_\xi + \xi = 0.$$

The following proposition presents the stabilizability condition

Proposition 4: For system (A_i, B), α_i, with $i \in \mathcal{I}$ such that the pair $(\sum_i \alpha_i A_i, B)$ is controllable. Then, the switched affine system is quadratically stabilizable.

We can find a feedback K_1 such that $A_0 + B_0 K_1$ is Hurwitz, and make $u_1 = u_2 = K_1 x$. The eigenvalues of the average matrix $A_0 + B_0 K_1$ can be arbitrarily (symmetrically) assigned by appropriately choosing K_1. In this work, we use the pole placement technique. Let us define $A_{K0} = A_0 + B_0 K_1$ as the average closed-loop matrix. As in Assumption 3, in closed-loop we have similar assumption

Assumption 5: There is a convex combination of $A_{Ki} = (A_i - BK_i)$ with $i \in \mathcal{I}$, which is Hurwitz.

We present a state-feedback switching signal based on an appropriate partition of the state space. Assuming (5) holds, we solve the Lyapunov equation

$$A_{K0}^\top P + P A_{K0} = -I_n,$$

with the additional constraint system

$$A_{K0} x_\xi + \xi_0 = 0 \tag{8}$$

All x_ξ satisfying (8) constitutes the equilibrium set for the closed-loop system

$$X_\xi^K = \{-A_{K0}^\top \xi_0\}$$

and we obtain a symmetric positive definite matrix P, this matrix can be seen as a common Lyapunov function for the switched affine system. For each subsystem we denote

$$Q_i = A_{Ki}^\top P + P A_{Ki}, \quad i \in \mathcal{I}. \tag{9}$$

which are positive definite matrices. The main result for the feedback switching algorithm is presented in the following theorem.

Theorem 6: Consider the switched affine system (4) and let $x_\xi \in \mathbf{R}^n$ be given. If Assumption 5 holds and there exists positive matrices Q_i as in (9) such that $V = (x - x_\xi)^\top P (x - x_\xi)$. Then the state switching control

$$\sigma(x) = \arg\min_{i \in \mathcal{I}} \{(x - x_\xi)^\top (Q_i(x - x_\xi) + 2P(A_K ix + \xi_i))\}, \quad (10)$$

and the feedback control

$$u(x) = -K_0(x - x_\xi),$$

where $A_{Ki} = A_i - BK_1$, asymptotically stabilize system (4).

Notice that $\arg\min$ stands for the index which attains the minimum among $\mathcal{I} = \{1, 2, \cdots, N\}$. We have presented the main theoretical results to be implemented in the activated sludge process modeled as a switched affine system (4). In the next section, some simulation results are presented to illustrate the implementation of Theorem 6 and particularly the switching control (10).

IV. Simulation Results

In order to implement the switching signal, the conditions of Proposition 4 have to be verified, and α_1 and α_2 have to be selected to obtain a convex combination. Here, the simple average combination, i.e., $\alpha_1 = \alpha_2 = 1/2$, is tested, with the pair (A_0, B) being controllable. Using Lemma 3 and applying the pole placement technique on the average closed-loop matrix A_{K0}, the feedback gains $K_1 = [-1.6345 - 0.2326 - 0.4966 - 2.1722]^\top \cdot 10^4$ are obtained. To validate the response of the proposed switching control, the main result presented in Theorem 6 is implemented in a MATLAB/Simulink simulator. In this case, $N = 2$ and $n = 4$. Three different weather conditions are considered, based on the influent data presented in [18]. The influent profile is shown in Fig. 2.

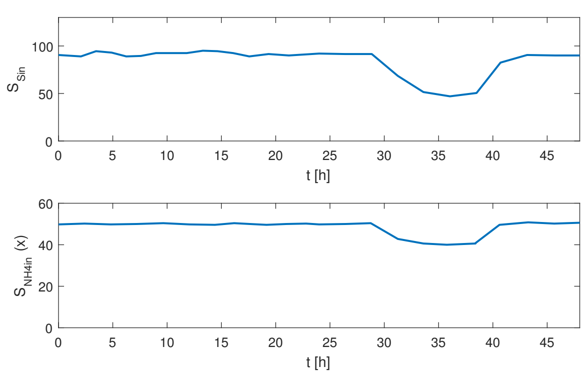

Fig. 2. Concentration of S_{Sin} and $S_{NH_{4in}}$ under dynamic weather changes

During the first 24 hours the influent presents variations in the concentration of S_{Sin} and S_{NH4in}, typical of dry weather. Then, at $t = 28h$ a storm event begins and lasts for 2 hours, followed by a rain event between $t = 30h$ until $t = 38h$. During these storm and rain events, a drop in the concentration of S_{Sin} and S_{NH4in} can be observed. After the rain stops, the concentrations of the influent restore to the values of the dry weather. The simulation consists of two scenarios in which the dynamic response of the four state variables are analyzed, i.e., $x = [S_S \; S_{NO_3} \; S_{NH_4} \; S_{O_2}]^\top \; g.m^{-3}$ with initial conditions $x(0) = [10 \; 8 \; 6.5 \; 0]^\top \; g.m^{-3}$. First, the continuous input control and the state-based switching signal, corresponding to "full control", is applied. Then, a constant switching signal is applied with a predefined constant control $u = S_{Sc} = 4000 \; g.m^{-3}$ (open-loop operation). Fig. 3 shows the evolution of the concentration of substrate S_S and concentration of Nitrate S_{NO_3} under these two scenarios. We can observe that full control drives the concentration of S_S to one of the equilibrium points and keeps the concentration lower as compared to open-loop operation (constant u). This affects also the concentrations of S_{NO_3} and S_{NH_4} (see Fig. 4). We can appreciate that in average the proposed control strategy keeps the concentrations of S_{NO_3} and S_{NH_4} at lower levels.

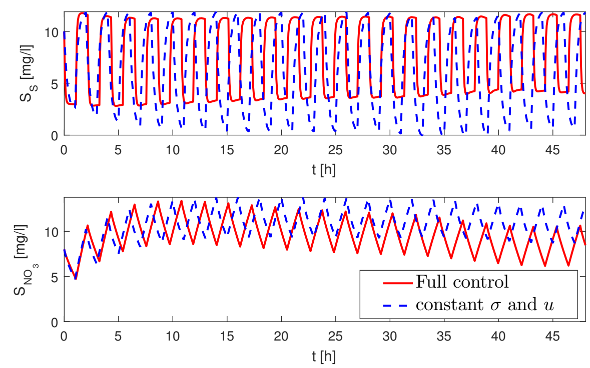

Fig. 3. Dynamic response of S_S and S_{NO_3} under full control and open-loop operation

Moreover, it is observed in Fig. 4 that the controlled concentration of oxygen S_{O_2} is lower under the proposed control strategy than the other scenario. Finally, the switching and feedback control signals are shown in Fig. 5, and it is noticed that the switching signal has a quasi-periodic form, which is also reflected in the feedback control law.

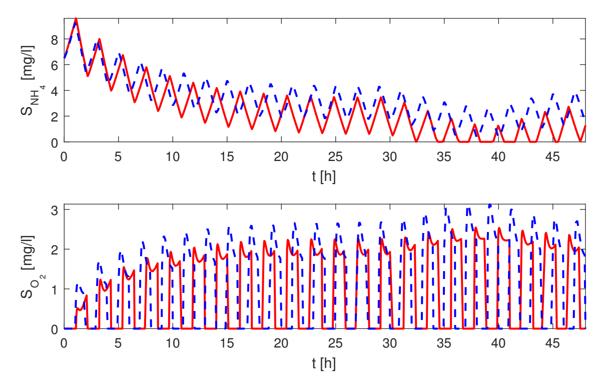

Fig. 4. Dynamic response of S_{NH_4} and S_{O_2} under full control and open-loop operation

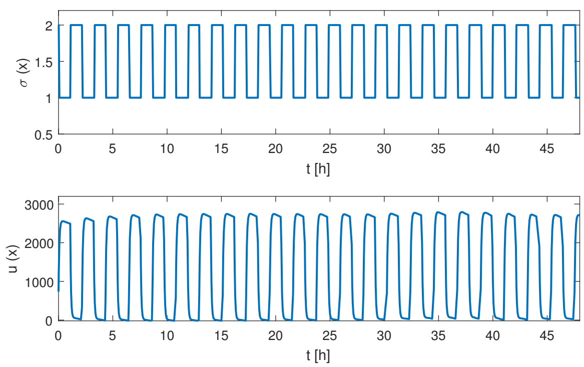

Fig. 5. Switching control signal $\sigma(x)$ and feedback control $u(x)$

V. Conclusion

In this paper, we have proposed a switched affine model of the continuous activated sludge process for control adapted from a previously two phase model. Some assumptions have been made to adapt the model and to include the control variables in a more traditional way. Based on this switched affine model, we develop and implement in simulation a state feedback switching control law that stabilize the closed-loop system to an equilibrium set. The main result is the integration of the feedback external input controlling the concentration of carbon and a state switching signal, all for a switched affine system. Some simulation allow us to observe the effectiveness of the proposed algorithm.

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