Time reflection and time refraction of graphene plasmons

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Abstract – Changing materials in time gives rise to a special type of reflection and refraction. Here we show that graphene plasmons propagating along a graphene sheet and crossing a temporal boundary experience reflection and transmission, resembling Fresnel reflection and transmission taking place at a spatial boundary. The temporal discontinuity we use is a change of Fermi level in the graphene sheet. The shape of the discontinuity can be tailored to filter specific frequencies. This phenomenon is fairly general and can be extended to other guided resonances.

I. INTRODUCTION

Graphene plasmonics has emerged as a substantial area of research because of numerous remarkable properties of graphene plasmons: they are highly tunable by controlling the graphene Fermi level, they exhibit a deeply subwavelength confinement and display a fairly low amount of losses. In another field of research, time-dependent phenomena have also recently been studied: they give rise to interband photonic transitions, wavelength conversion and topological states of light. The behaviour of electromagnetic waves at a time boundary has been studied since 1958 [1] and has been extended upon since in recent works [2, 3]. However, to the best of our knowledge, the behaviour of guided modes incident on a time boundary has not been described yet. We choose graphene plasmons for this study because of their tunability [4, 5], but the same analysis can be generalized to other guided modes such as waveguides for example.

II. GRAPHENE CONDUCTIVITY AND GRAPHENE PLASMONS

In this paper we study the behaviour of graphene plasmons at a temporal discontinuity. Graphene is in its natural undoped state a zero gap semiconductor but can be turned into a metallic layer by changing its Fermi level $E_F$. This can be done by chemical doping or by electrostatic gating [6]. Graphene is described in theory and in finite element method (FEM) simulations by a current line with a Drude-like conductivity [7]:

$$\sigma(\omega) = \frac{e^2 E_F}{\pi h^2} \frac{-j}{\omega - j \frac{1}{\tau_{\text{gra}}}}$$

This equation is valid for $E_F \gg k_B T$, with $k_B T \approx 0.026 \text{ eV}$ at room temperature. The term $\frac{1}{\tau_{\text{gra}}}$ accounts for electron scattering and is responsible for losses of graphene plasmonic modes. Since we want to study propagation of graphene plasmons, we will use $\frac{1}{\tau_{\text{gra}}} = 0$ for convenience, allowing graphene plasmons to propagate without losses. Even though this value is not physical, the effects presented in this paper still remain even with losses because the results we give do not depend on the intensity of the plasmonic mode. Using this conductivity expression, one can derive the dispersion relation of graphene plasmonic modes in the nonretarded regime [8] ($\beta \gg \omega/c$ where $\beta$ is the plasmonic mode propagation constant):

$$\text{Re}(\beta) = \frac{2\varepsilon_0 \varepsilon_r \pi h^2 \omega^2}{e^2 E_F}$$

By making the Fermi level a function of time, one can change the graphene conductivity and in turn affect the plasmonic modes.
Recently, it was shown that the proper boundary conditions at a temporal interface are [2]:

\[ D(t = 0^-) = D(t = 0^+), \quad B(t = 0^-) = B(t = 0^+) \]  

that impose the continuity of the electric displacement \( D \) and magnetic induction \( B \) across the time boundary. In the case of a space boundary, \( \omega \) is conserved on both sides of the boundary. In the temporal case, it is the propagation constant that is conserved across the temporal boundary [9]. Consequently, in a dispersive medium, the frequency of the considered mode has to change to match that condition. If we consider a graphene plasmon incident on a time boundary step profile with the propagation constant \( \beta_i \) of (2), a transmitted and reflected plasmon pulse will be produced with propagation constants \( \beta_t \) and \( \beta_r \), respectively. The boundary conditions [9, 10] impose that

\[ \beta_i = \beta_t = \beta_r. \]  

Using the dispersion relation of 2, this condition can be rewritten as:

\[ \frac{\omega_i}{\sqrt{E_{F0}}} = \frac{\omega_t}{\sqrt{E_{F1}}} = -\frac{\omega_r}{\sqrt{E_{F1}}} \]  

where \( E_{F0} \) and \( E_{F1} \) are the Fermi level before and after the temporal interface, respectively. The minus sign preceding the \( \omega_r \) term is there because the reflected wave propagates backward in space. Then by imposing the continuity of (3) and of its time derivative, one can obtain the Fresnel time reflection coefficients of graphene plasmons at a temporal interface:

\[ T_{\text{step}} = \left( \frac{\delta + 1}{2\delta} \right)^2, \quad R_{\text{step}} = \left( \frac{\delta - 1}{2\delta} \right)^2 \]  

where \( \delta = \sqrt{E_{F1}/E_{F0}} \). When using a slab instead of a step time profile, the transmission and reflection Fresnel coefficients become:

\[ T_{\text{slab}} = 1 + \kappa \sin^2 (\delta_{12} \omega \tau), \quad R_{\text{slab}} = (\delta_{12} - \delta_{12})^2 \sin^2 (\delta_{12} \omega \tau), \]  

where \( \kappa = 1/4 \left( \delta_{12} + \delta_{12} \right)^2 - 1, \delta_{12} = \sqrt{E_{F1}/E_{F0}}, \delta_{21} = \sqrt{E_{F0}/E_{F1}}, \tau \) is the slab time duration and \( \omega \) is the incident plasmonic mode angular frequency. Note that unlike the spatial case, the reflected wave cannot interact with previous time boundaries because of causality.

**IV. COMPARISON WITH SIMULATIONS**

To validate the results presented in the previous section, we run FEM simulations with COMSOL in the time domain of a graphene plasmon interacting with a temporal slab. The profile of the \( H \) field of the plasmonic mode (graphene plasmons are TM modes) at different times is shown in Figure 1 where one can see the generation of a reflected pulse after the time interface.

Fig. 1: FEM simulation results for the \( H \) field profile of the plasmonic mode along the propagation direction (left) before (middle) during and (right) after the slab time boundaries. The time slab as well as the notation used is also pictured on the far right of the figure.
We also compare equations (7) to FEM simulations in Figure 2 for different time slabs profiles (with different $E_{F1}$). The simulations results are in good agreement with the theory.

![Figure 2](image_url)

**Fig. 2:** (Dots) Results from Eqs. (7) and (solid lines) comparison with FEM simulations for different time slab heights. We used a slab length of $\tau = 1.06 \times 10^{-14}$ s and $E_{F0} = 0.6$ eV for all the curves.

V. CONCLUSION

We computed the time reflection and refraction Fresnel coefficients of graphene plasmons at a time interface using a theoretical model based on the continuity of electric displacement and magnetic induction at a time boundary. These results are in good agreement with FEM simulations. The methods used in this paper can be used to describe other guided modes (in waveguides for example) at a temporal interface.

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